Competition, Bonuses, and Risk-taking in the Banking Industry*

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Abstract

Remuneration systems in the banking industry, in particular bonus payments, have frequently been blamed for contributing to the build-up of risks leading to the recent financial crisis. In our model banks compete for managerial talent that is private information. Competition for talent sets incentives to offer bonuses inducing risk-taking that is excessive not only from society's perspective, but also from the viewpoint of the banks themselves. In fact, bonus payments and excessive risk-taking are increasing with competition. Thus, our model offers a rationale why bonuses are paid even when reducing the expected profits of banks.

JEL Classification: D86, J31, J33

Keywords: Bonuses, excessive risk, screening, competition

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"Banks operate in a world where their star talent is apt to jump between different groups, whenever a bigger pay-packet appears, with scant regard for corporate loyalty or employment contracts. [...] As one banker says: 'These bonuses are crazy - we all know that. But we don't know how to stop paying them without losing our best staff.' "
(Tett, 2009)

1 Introduction

Payments in the banking industry have risen tremendously in the decade leading up to the financial crises 2007/08, triggering intensive debates on the optimal level and structure of compensation schemes. Suntheim (2010) examines CEO compensation in the financial sector and finds that in a sample of 74 banks from 18 countries average CEO payment increased from \$2,332,944 in 1997 to \$5,368,365 in 2007. Slightly more than half of this sum (55%) has been paid as bonuses. Below the executive rank, the bonus proportion of compensation often tends to be even higher, particularly among traders and sales persons (OECD, 2009; Murphy, 2009).

Frequently, compensation systems in the banking industry have been blamed for causing or at least contributing to the build-up of risks that led to the eruption of the recent credit crisis (Bebchuk et al., 2010; Berndt et al., 2010). In particular, the sheer size of bonus payments has led to popular outrage, reflecting the connotation of bonuses as short-term incentives that trigger excessive risk-taking (Taleb, 2009).

Most industry experts agree that two underlying incentive problems can be distinguished. A first agency problem exists between banks' shareholders and top executives on the one hand and society on the other hand: Due to limited liability, induced by deposit insurance and by likely bail-outs via the tax payers, banks have an incentive to deliberately take risks that increase expected profits but reduce social welfare (Flannery, 1998; Sironi, 2003; Gropp et al., 2011).

The second agency problem, in contrast, arises between banks' shareholders and CEOs on the one hand and employees such as traders, investment bankers, loan officers or mortgage brokers on the other hand (Clementi et al., 2009). Our paper builds upon the latter, internal agency issue. At its core, it implies that banks' remuneration systems induce risk-taking *on top* of what may be desired by the banks' owners themselves. In particular, so the argument, inadequate risk adjustments for returns have led employees to create excessively high risks that are not in the banks' interest, even when taking into account that the banks and their shareholders benefit from limited liability (Rajan, 2008; Taleb, 2009; OECD, 2009).

If compensation packages in the banking industry indeed induce too much risk, the question arises why such sub-optimal payment systems are offered in the first place, i.e., why banks pay bonuses that lead to risks "that may either increase or decrease the value of the bank's assets, but whose expected effect on the bank's value is negative" (Bebchuk and Spamann, 2010). We suggest competition for managerial talent as a main explanatory factor. To this end, we consider a model where agents of different talent (or ability types) have to decide on the allocation of funds between safe and risky projects. An agent's type influences the returns from both safe and risky investments, but is unobservable, i.e., private information. The agents' remuneration depends on the realized return, while the portfolio choice itself is non-contractible. Hence, our model combines features of hidden information on talent, moral hazard with respect to project choice, and competition for workers.

Workers' talent is modelled in the simplest way by assuming that high-ability workers generate higher (expected) returns both from safe and from risky projects. We then derive two results: First, we show that a separating equilibrium exists under fairly plausible conditions and that bonuses are only offered in the contract designed for the high-ability type. The separating equilibrium arises because the high type's marginal rate of substitution between fixed and variable (i.e., bonus) compensation is greater than that of the low-ability type as she generates a higher expected return. As a consequence, bonus components in wage contracts can be used as a screening device, and the bank deliberately accepts the inefficiencies from the high type's overly risky investment strategy induced by the bonus offering in order to reduce the low-ability type's information rent.

Second, and in our view most importantly, we find that bonuses - and hence also the degree of excessive risk-taking and inefficiency - are increasing in the degree of competition for workers. Rising competition implies that each bank must offer higher expected total compensation to be able to attract a worker. Given her superior ability, this effect is particularly pronounced for the high type. Unfortunately, the increase in expected total compensation automatically raises the information rent the low type can obtain from imitating the high type – provided the composition of the high type's compensation, i.e., the division into fixed wage and bonus component, remains constant. In order to curb the low type's imitation incentive and reduce her information rent, the bank increases the bonus fraction in the high type's contract as competition becomes fiercer, even though this induces even further excessive risk-taking. When competition for workers decreases, by contrast, the bank needs to pay less to the high type. This reduces the low type's imitation incentive, and the bank is therefore no longer willing to deliberately accept the inefficiencies from excessive risk-taking that arise from bonuses. Hence, competition for workers leads not only to higher overall salaries, but also to higher bonuses, higher risk-taking and higher inefficiencies both from the society's and the bank's point of view.

In our main model, we restrict attention to non-negative bonuses, which implies a strong form of limited liability. A natural question is whether the induced inefficiencies can be mitigated by demanding a co-investment from agents. We find that these payments, designed as negative bonuses when returns are below the return of safe projects, indeed reduce the risk taken by the high type.[1] While this assigns an important role to

[1]We are thankful to an anonymous referee for pointing out this aspect.

co-investments of risk-taking units in banks, we show that these payments resolve the problem of excessive risk-taking only if the agent's wealth is sufficiently high.

The main result of our paper is that fiercer labour market competition aggravates the problem of excessive risk-taking by increasing the agents' outside options. Although their framework is quite different, this is related to findings by Stoughton and Zechner (2007). In their model, the bank allocates its capital to different division managers who have private information. It is shown that the second-best optimal capital allocation may be implemented via EVA and RAROC-based compensation schemes. Due to the managers' private information, this second-best induces excessive risk-taking, and as in our model, excessive risk-taking becomes more pronounced when managers' outside options improve.

In our model, bonuses are offered merely to attract talented managers – which becomes more difficult the fiercer the competition for these high-ability managers is – but do not induce any positive real effects. As such, our paper assigns a positive role to regulating pay in the banking sector: Setting a regulatory cap on bonus payments could drive a wedge between competitive pressures to attract talented bankers on the one hand and the unintended risk-taking incentives induced by their compensation packages on the other hand. Of course, we are aware of the fact that our approach neglects potentially beneficial effects of bonuses such as increasing the incentives to foster managerial talent (Kashyap et al., 2008) and to exert higher effort in identifying promising investment opportunities.[2] The potential impact of these countervailing effects on our model are discussed in a section on robustness.

Although the paper is purely theoretical, some remarks on empirical research are in order. Overall, the evidence on compensation-induced risk-taking is mixed. Several empirical studies have indeed confirmed a positive correlation between bonuses and equity/option compensation on the one hand and risk-taking on the other hand (Cheng et al., 2011; DeYoung et al., 2010). Suntheim (2010) shows that banks offering high risk-taking incentives to their managers perform worse than banks that offer a larger fraction of pay via base salaries. Other studies, however, report no clear-cut evidence for the role of compensation systems in the financial crisis. Fahlenbrach and Stulz (2011) find no evidence that banks' compensation packages had any detrimental effect with regard to the onset of the credit crisis, and argue that bank CEOs faced high losses from the financial crisis. Bebchuk et al. (2010), however, contradict this view by pointing out that the salaries prior to the crisis have largely overcompensated the subsequent losses. Balachandran et al. (2010), finally, observe a harmful impact of equity-based pay but not of non-equity incentive pay. It should be noted, however, that most empirical studies so far analyze only CEOs' remuneration schemes and not the pay packages of lower-ranked bank employees who are in the focus of our model. Changes in compensation practices following the financial crisis, by contrast, appear to

[2] Dittman and Yu (2009) even indicate that any risk-taking incentives in payment schemes may be offered inadvertently, possibly only as a side effect of effort incentives.

tackle particularly this hierarchical segment by introducing e.g. bonus-malus-schemes or deferred bonus payments for high-performing employees below the executive board level (Clementi et al., 2009).

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 focuses on the agent's portfolio choice. Section 4 derives the optimal compensation package offered by the banks, while section 5 considers the impact of competition. Section 6 discusses the robustness of our modelling assumptions. Section 7 relates our paper to the literature and section 8 concludes.

2 The Model

Banks, agents and projects. We consider a model with two banks, $i \in \{G, B\}$ competing for an agent. The agent's type $k \in \{H, L\}$ is private information, and is H (high) with probability α and L (low) with probability $1-\alpha$. When hired by bank i, the agent invests a fixed amount normalized to 1 into a portfolio consisting of a fraction $\gamma_k^i \in [0,1]$ of a risky project and a fraction $1-\gamma_k^i$ of a safe project. The agent's project choice is uncontractible so that her bonus can only depend on the return actually realized.

The net return of the safe project is θ_k^i. For the risky project, the net return is $\xi_k^i \in (-1, \infty)$, where ξ_k^i is an $\mathcal{F}$-measurable random variable on a probability space $(\Omega, \mathcal{F}, \mathbf{P})$, with $\mathbb{E}|\xi_k^i| < \infty$. We assume $\theta_k^i > \mathbb{E}\xi_k^i > 0$, i.e., the expected return of the risky asset is positive but strictly below the return of the safe one. Of course, assuming that the risky project has lower expected return contradicts the usual risk-return trade-off, but captures in the most simple way that safe projects are superior. We can then assume that all parties are risk-neutral, and we can talk of excessive risk-taking whenever $\gamma_k^i > 0$. Our main results also hold when assuming that the risky project has higher expected return and banks are risk-averse, but this leads to a much more involved model without yielding additional insights (see our discussion in section 6.3).

We assume that, in both banks, the high agent type is more productive than the low agent type. For the safe project, this means that $\theta_H^i > \theta_L^i > 0$, for each $i \in \{G, B\}$. For the risky project, the high type's productivity advantage is captured by assuming that ξ_H^i first-order stochastically dominates ξ_L^i, that is, $\mathbf{P}(\xi_H^i \leq x) \leq \mathbf{P}(\xi_L^i \leq x)$, for all $x \in \mathbb{R}$, and the inequality is strict for some $x \in \mathbb{R}$. We also assume that $\mathbb{E}\left[(\xi_H - \theta_H)^+\right] \geq \mathbb{E}\left[(\xi_L - \theta_L)^+\right]$, where we have used the short-hand notation $x^+ := \max(x, 0)$. This expresses that the productivity difference of the positive part – which will be relevant for bonus payments – of the two types is weakly greater for the H-type agent, and ensures that in equilibrium the L-type agent takes less risk than the H-type agent.

Next, we assume that both agent types generate higher returns in the good bank G than in the bad bank B. Specifically, we assume that there exists some $\beta \in [0, 1]$ such that $\theta_k^B = \beta\theta_k^G$ for the safe project and $\mathbb{E}\xi_k^B = \beta\mathbb{E}\xi_k^G$ for the risky project, for both agent types $k \in \{H, L\}$. As we shall see, differentiating between the expected returns in the two banks is a convenient way of modelling the degree of competition for agents. This labour market competition is important for the contracts offered in equilibrium. If an agent is equally productive in both banks, then we have perfect competition for her. Most naturally, β might also be interpreted as the degree of specificity of human capital as this determines the productivity difference of workers in different firms.

After having accepted an offer of bank i, agent k decides on the fraction γ_k^i invested in the risky project. The agents' effort costs depend on the fraction invested in the risky project and are given by $C(\gamma)$ with $C(0) = 0$, $C'(\gamma) > 0$, $C''(\gamma) > 0$.[3] For technical ease we assume that effort costs are increasing fast enough to ensure that the agent always chooses $\gamma < 1$.

We assume that banks have deep pockets, i.e., they face no bankruptcy risk. This ensures that banks prefer the safe project due to its higher expected return. If there were positive bankruptcy risk, banks themselves might prefer risky projects despite their lower expected return, since they could then benefit from the upside while externalizing part of the downside risk to third parties. This is excluded as we are interested in whether competition for agents induces excessive risk-taking even from the banks' point of view, and not only from a social perspective.

Payoffs. For notational convenience we omit the index of the bank. If an agent of type k invests γ in the risky and $1 - \gamma$ in the safe asset, then after one time period the portfolio is worth

$$\gamma(1 + \xi_k) + (1 - \gamma)(1 + \theta_k) = 1 + \theta_k + \gamma(\xi_k - \theta_k),$$

with a net gain of $\theta_k + \gamma(\xi_k - \theta_k)$.

We restrict attention to linear bonuses and assume that these are only paid if the return exceeds the threshold θ_H, which is the net return the high type can generate with the safe project. With a fixed wage of F and a bonus fraction $w \in [0, 1]$, the agent's utility is

$$F + w\left(\theta_k + \gamma\left(\xi_k - \theta_k\right) - \theta_H\right)^+ - C(\gamma).$$

Finally, the bank's profit from agent k is

$$\theta_k + \gamma(\xi_k - \theta_k) - F - w\left(\theta_k + \gamma\left(\xi_k - \theta_k\right) - \theta_H\right)^+.$$

[3] Assuming that effort costs are increasing in γ can be attributed to different problems the agent faces when choosing the risky project. For instance, risky projects that are in the agent's interest must be identified, while investing in safe projects may be simpler. Moreover, the agent might need to camouflage that she has invested in projects that are not entirely in the bank's interest. In any case, it is realistic that the necessary effort is higher for risky projects.

Competition for agents. Banks compete for an agent by simultaneously offering take-it-or-leave-it contracts from a set $\mathbf{\Omega}^i$, $i \in \{G, B\}$, where a contract $(F, w) \in \mathbf{\Omega}^i$ is specified by a fixed wage F and a bonus fraction $w \in [0, 1]$. The expected utility of contract (F, w) proposed by bank i to an agent of type k is given by

$$U_k^i(F, w) = \max_{\gamma \in [0,1]} F + w \, \mathbb{E}\left[\left(\theta_k^i + \gamma(\xi_k^i - \theta_k^i) - \theta_H^i\right)^+\right] - C(\gamma).$$

We define $\widehat{U}_k^i$ as the maximum expected utility agent k can get from bank i, that is,

$$\widehat{U}_k^i := \max_{(F,w) \in \mathbf{\Omega}^i} U_k^i(F, w),$$

and clearly the agent will choose a contract with expected utility $\widehat{U}_k^i$. To simplify the exposition, we introduce the tie-breaking rule that both types accept the good bank's offer if $\widehat{U}_k^G = \widehat{U}_k^B$.

Sequence of events. Since the agent's type is private information and as the two banks compete simultaneously for her, the game structure is as follows:

- *Stage 0*: Nature chooses the agent's type which becomes private information.
- *Stage 1*: Banks simultaneously offer take-it-or-leave-it contracts to the agent.
- *Stage 2*: Depending on her type, the agent chooses her utility-maximizing contract.
- *Stage 3*: The agent decides upon the portfolio risk expressed by γ.
- *Stage 4*: Returns and payments are realized.

Example. The combination of asymmetric information on types, moral hazard on risk-taking and competition for agents introduces a certain level of complexity to the model. We therefore illustrate the main properties of the model numerically and graphically by a fully worked-out example. In the example, we assume that returns are lognormally distributed. For the good bank – omitting the index G – the discrete returns of the risky project are given by

$$\xi_k = \mathbf{e}^{\mu_k - \frac{\sigma^2}{2} + \sigma X} - 1, \qquad k \in \{H, L\},$$

where X is a standard normally distributed random variable, μ_k is such that $\mathbb{E}\xi_k = \mathbf{e}^{\mu_k} - 1 < \theta_k$, and $\mu_H \geq \mu_L$. In other words, $1 + \xi_k$ follows a lognormal distribution with parameters $\left(\mu_k - 1/2\sigma^2, \sigma^2\right)$. The condition $\mu_H \geq \mu_L$ together with the common risk parameter σ implies that ξ_H stochastically first-order dominates ξ_L. The parameters of the example are given in Table i. Figure i shows the densities of the risky project returns ξ_k in the good bank and the expected net gains $\theta_k + \gamma(\xi_k - \theta_k)$ as a function of the portfolio choice γ.

Parameter	Value
θ_H, θ_L	$0.15, 0.07$
$\mathbb{E}\xi_H, \mathbb{E}\xi_L$	$0.08, 0.02$
σ	0.325
α	0.15
β	0.9
Effort function $C(\gamma)$	$0.0558\,\gamma^2$

Table I: Example parameters

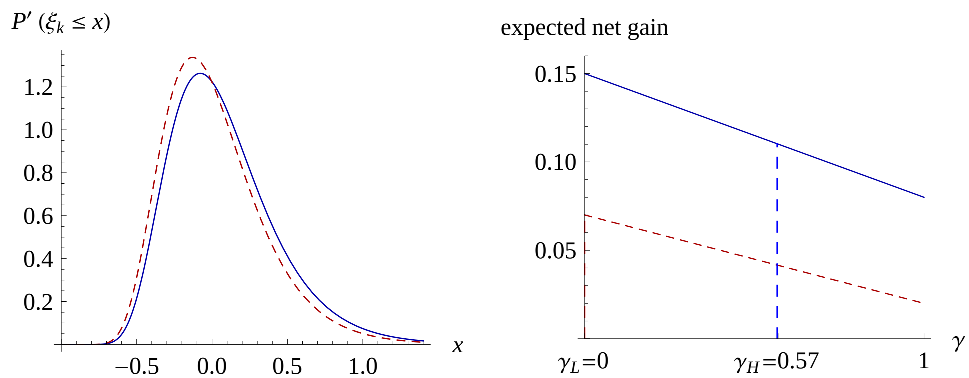

Figure 1: Densities of risky project return (left) and expected net gains (right). Solid line: H-type agent, dashed line: L-type agent. For future reference, γ_H denotes the portfolio choice of the H-type agent in equilibrium, and γ_L denotes the respective choice of the L-type agent.

3 The Agent's Portfolio Choice

Following backwards induction, we start with the agent's portfolio choice in stage 3. The agent chooses a fraction of the risky asset that maximizes her expected utility, that is,

$$\gamma_k^i(w) = \underset{\gamma \in [0,1]}{\operatorname{argmax}}\, F + w\,\mathbb{E}\left[\left(\theta_k^i + \gamma(\xi_k^i - \theta_k^i) - \theta_H^i\right)^+\right] - C(\gamma). \tag{1}$$

For our example, the agents' choices of γ_H, γ_L in the good bank and their utilities derived from the bonus component are shown in Figure ii.

We shall frequently use the following Lemma:

LEMMA 1. *(i) As the bonus fraction w increases, an agent's appetite for the risky asset increases, that is,* $\frac{\partial}{\partial w}\gamma_k^i(w) \geq 0$.

(ii) The high type benefits more from an increase in the bonus than the low type, that

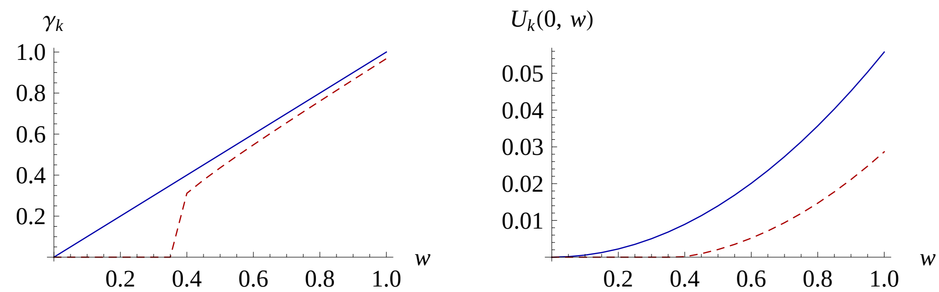

Figure 2: Left: Agents' portfolio choices γ_H (solid) and γ_L (dashed); right: agents' utilities derived from bonus component, $U_H(0, w)$ (solid) and $U_L(0, w)$ (dashed).

is,

$$\frac{\partial}{\partial w}\left(U_H(F, w) - U_L(F, w)\right) > 0, \qquad \textit{for all } F \geq 0 \textit{ and } w \in [0, 1]. \tag{2}$$

Proof. See Appendix A. □

The agent chooses a fraction of the risky project, despite its lower expected return when compared to the safe project, as the bonus is non-negative. In other words, the agent benefits from the uncertainty of the realized return, as she is not held liable for losses.

In stage 2, the agent will simply accept the contract that maximizes her expected utility, thereby anticipating that she will choose the fraction $\gamma_k^i(w)$ as given by Equation (1). Hence, we can directly proceed to the contracts offered by banks in stage 1.

4 The Good Bank's Best Response Function

Assume that bank B offers a set of contracts fulfilling $\widehat{U}_H^B \geq \widehat{U}_L^B$, i.e., the maximum expected utility the high type can obtain from the contracts offered by the bad bank is greater than or equal to the low type's. This assumption clearly holds in equilibrium as the high type can always choose the contract designed for the low type, and can never get lower expected utility due to the first-order stochastic dominance property of ξ_H^B over ξ_L^B.

In the following, we analyze the good bank's best response to $(\widehat{U}_H^B, \widehat{U}_L^B)$ in a separating equilibrium. We show later that there can be no equilibrium in which one or both banks offer a pooling contract.

To ease notation, we omit the index G whenever it is clear that we refer to bank G.

The bank's optimization problem is

$$\max_{F_H,F_L,w_H,w_L} \alpha\,\mathbb{E}\left[\theta_H + \gamma_H(w_H)(\xi_H - \theta_H) - F_H - w_H\,\gamma_H(w_H)(\xi_H - \theta_H)^+\right]$$
$$+ (1-\alpha)\,\mathbb{E}\left[\theta_L + \gamma_L(w_L)(\xi_L - \theta_L) - F_L - w_L(\theta_L + \gamma_L(w_L)(\xi_L - \theta_L) - \theta_H)^+\right], \quad (3)$$

with $\gamma_k(w)$ given by Equation (1), and subject to the constraints

(i) $U_H(F_H, w_H) \geq U_H(F_L, w_L)$ *(ICCH)*,

(ii) $U_H(F_H, w_H) \geq \widehat{U}_H^B$ *(PCH)*,

(iii) $U_L(F_L, w_L) \geq U_L(F_H, w_H)$ *(ICCL)*,

(iv) $U_L(F_L, w_L) \geq \widehat{U}_L^B$ *(PCL)*.

The first (last) two restrictions refer to the high (low) type. In addition, we have the technical constraint $0 \leq w_H, w_L \leq 1$.

The following Lemma simplifies the optimization problem:

LEMMA 2. *In the optimal menu of contracts offered by the good bank, (i) the bonus for the low type is zero; (ii) if separating contracts are offered, the high type's incentive compatibility constraint, (ICCH), is non-binding; (iii) the high type's participation constraint, (PCH), is binding; (iv) the low type's incentive compatibility constraint, (ICCL), is binding.*

Proof. See Appendix B. □

For *part (i)*, note that the low type invests in the safe project if the bonus is zero. As this is optimal for the bank, it will pay a fixed wage only.

Part (ii) mirrors the standard result that the high type has no incentive to imitate the low type. Hence, the high type's incentive compatibility constraint *(ICCH)* is slack, and it follows immediately that the bank will reduce the high type's fixed wage F_H until the participation constraint *(PCH)* becomes binding. This explains *part (iii)*.

Concerning part *(iv)*, note that the low type's incentive to imitate the high type is decreasing when the bank puts more weight on bonuses compared to fixed wages.[4] This, however, is costly as it increases the fraction γ the high type invests in the risky project. Hence, the bank will choose the lowest bonus that only just prevents the low type from imitating, and *(ICCL)* will be binding.

As a consequence of Lemma 2, the optimization problem Equation (3) reduces to a problem involving only the variable w_H, which we denote by w from now on. Using that *(PCH)* and *(ICCL)* are binding, we write

$$F_H =: F_H(w) = \widehat{U}_H^B - U_H(0, w)$$
$$F_L =: F_L(w) = F_H(w) + U_L(0, w),$$

[4] This is just a reformulation of Equation (2).

and we rewrite the optimization problem as follows:

$$\max_w L(w) = \alpha \left\{ \mathbb{E}\left[\theta_H + \gamma_H(w)(\xi_H - \theta_H)\right] - \widehat{U}_H^B - C(\gamma_H(w)) \right\}$$
$$+ (1-\alpha) \left\{ \theta_L - \widehat{U}_H^B + U_H(0,w) - U_L(0,w) \right\}, \tag{4}$$

subject to the constraint *(PCL)*, which may be written as $U_H(0,w) - U_L(0,w) \leq \Delta\widehat{U}^B$, where $\Delta\widehat{U}^B := \widehat{U}_H^B - \widehat{U}_L^B$. As before, $\gamma_H(w)$ is given by Equation (1).

In the following, we assume that the density function of the risky return and the effort function are such that L has exactly one maximum on $(0,1)$. The bonus fraction that solves the optimization problem is denoted by w^*, that is, $w^* := \operatorname{argmax}_w L(w)$ subject to the constraint *(PCL)*.

Having reduced the problem to an optimization problem involving only w, it remains to determine the optimal "mix" of $w, F_H(w), F_L(w)$. It turns out that this is closely linked to $\Delta\widehat{U}^B := \widehat{U}_H^B - \widehat{U}_L^B$, which in turn determines whether *(PCL)* is binding or not.

PROPOSITION 1. *The bonus in the high type's contract in the good bank's best response function depends on the difference in the utilities the two agent types get in the bad bank, $\Delta\widehat{U}^B := \widehat{U}_H^B - \widehat{U}_L^B$. For w^*, two regions can be distinguished:*
Region 1*: If $\Delta\widehat{U}^B$ is above some threshold $\Delta\widehat{U}_T^B$, then the low type's participation constraint* (PCL) *in the good bank's best response function is non-binding, and the bonus w_1^* is implicitly given by the first-order condition*

$$\frac{\partial L}{\partial w} = \alpha\mathbb{E}\left[\gamma_H'(w_1^*)(\xi_H - \theta_H)\right] - \alpha C'(\gamma_H(w_1^*))\,\gamma_H'(w_1^*)$$
$$+ (1-\alpha)\left\{ \frac{\partial}{\partial w}\left(U_H(F_H(w_1^*), w_1^*) - U_L(F_H(w_1^*), w_1^*)\right) \right\} = 0.$$

Furthermore, w_1^* is constant in region 1.
Region 2*: If $\Delta U^B \leq \Delta\widehat{U}_T^B$, then* (PCL) *is binding in the good bank's best response function, and the bonus w_2^* is implicitly given by*

$$w_2^*\mathbb{E}\left[\gamma_H(w_2^*)(\xi_H - \theta_H)^+ - (\gamma_L(w_2^*)(\xi_L - \theta_L) - (\theta_L - \theta_H))^+\right]$$
$$+ C(\gamma_L(w_2^*)) - C(\gamma_H(w_2^*)) = \Delta\widehat{U}^B. \tag{5}$$

Furthermore, w_2^* is strictly increasing in $\Delta\widehat{U}^B$ and $w_2^* \leq w_1^*$.

Proof. See Appendix B. □

As in region 1 the low type's participation constraint is non-binding, she receives a positive information rent. In this case, the good bank faces the usual trade-off when designing the contract for the high type: On the one hand, increasing the bonus increases the fraction invested in the risky project, thus reducing profits when actually meeting

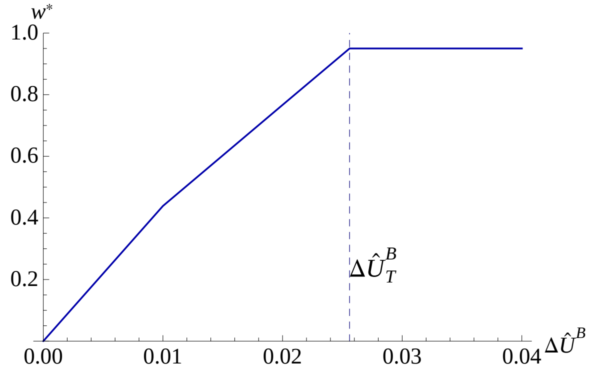

Figure 3: Optimal bonus fraction in the two regions.

the high type. On the other hand, a higher bonus reduces the low type's imitation incentive and thus the fixed wage F_L required to fulfill her incentive compatibility constraint *(ICCL)*. This increases profits when actually meeting the low type. The first order condition in region 1 balances this trade-off at the margin.

On the other hand, if the low type's participation constraint is binding, then increasing the bonus does not allow to reduce the low type's fixed wage as she does not get an information rent, anyway. This is the case in region 2 where the low type's fixed wage, F_L, is not determined by her *(ICCL)*, but by her *(PCL)*. This sets an upper bound on the bonus as the only reason to pay bonuses is to reduce the low type's information rent. In other words, the bank can no longer balance the high type's excessive risk taking and the low type's information rent at the margin as this would violate the low type's participation constraint. Consequently, the bonus in region 2 is smaller than in region 1, $w_2^* \leq w_1^*$.

In the example, the threshold that separates the two regions turns out to be $\Delta\widehat{U}_T^B = 0.026$. Thus, if the difference in the utilities the two types can get in the bad bank is smaller than 0.026, then the bonus in the contract designed for the high type does not depend on the trade-off between rent reduction and efficiency loss, but simply on the low type's participation constraint. This is also illustrated in Figure iii: Up to $\Delta\widehat{U}_T^B = 0.026$, the bonus is increasing in $\Delta\widehat{U}^B$, while remaining constant for greater $\Delta\widehat{U}^B$. The greatest bonus fraction offered is $w^\star = 0.95$.

It remains to explain why *(PCL)* is binding if and only if $\Delta\widehat{U}^B$ is *below* some critical threshold denoted by $\Delta\widehat{U}_T^B$ in Proposition 1. Why does the utility *difference* the two agent types get from bank B determine the bonus offered by bank G? As $\Delta\widehat{U}^B = \widehat{U}_H^B - \widehat{U}_L^B$, *(PCL)* is likely to be binding when $\widehat{U}_H^B$ is low and when $\widehat{U}_L^B$ is high. The impact of $\widehat{U}_L^B$ is straightforward: If $\widehat{U}_L^B$ is high, then the fixed wage needs to be high to ensure that $F_L \geq \widehat{U}_L^B$ holds.

The impact of $\widehat{U}_H^B$ is more subtle, though: For any bonus given, the fixed wage F_H

bank G must offer to meet the agent's participation constraint *(PCH)* is increasing in $\widehat{U}_H^B$. But the higher F_H, the higher is ceteris paribus the low type's fixed wage F_L to prevent her from imitating, i.e. to satisfy *(ICCL)*. Hence, F_L is ceteris paribus increasing in $\widehat{U}_H^B$, which makes the low type's participation constraint $F_L \geq \widehat{U}_L^B$ less critical. This explains why the two regions depend on $\Delta\widehat{U}^B$, and not only on $\widehat{U}_L^B$.

5 The Impact of Competition

The last section has shown how the bonus offered by the good bank depends on the two types' utility difference $\Delta\widehat{U}^B$ in the bad bank. It remains to analyze how this utility difference itself depends on the banks' degree of competition for agents, β.

Recall from the model section that the (expected) returns the agents generate in the bad bank are $\theta_k^B = \beta\theta_k$ for the safe project and $\mathbb{E}\xi_k^B = \beta\mathbb{E}\xi_k$ for the risky project, respectively. If $\beta = 0$, then bank G has monopsonistic labour market power. By contrast, we have perfect competition for workers when $\beta = 1$. Hence, β conveniently measures the degree of competition for agents. The higher is β, the higher are the returns the two types generate in the bad bank and this increases, ceteris paribus, also the difference in their returns. As this should be reflected in the difference of their payments, we expect $\Delta\widehat{U}^B$ to be increasing in β. In the following, we show that the bonus offered to the high type is hence also increasing in β.

Owing to the interdependencies of the banks' equilibrium offers, the analysis is not trivial, and we defer the full formal derivation of the bad bank's best response function and of the equilibrium contract offers to Appendix C. Here, we sketch the model behavior in an intuitive way, and we shall make use of some straightforward properties that hold in equilibrium. This allows to exclude several parameter constellations and simplifies the exposition considerably.

Note first that, without equilibrium selection, there are infinitely many (implausible) equilibria. To see this, suppose the bad bank offers each type more than their total expected output. The bad bank would then face losses when attracting an agent, but as long as both types prefer the good bank's offers, the bad bank has no incentive to deviate. We exclude these equilibria by the following standard equilibrium selection criterion.

ASSUMPTION 1 (Equilibrium selection)**.** *Weakly dominated strategies are excluded.*

The first result then shows that only separating equilibria can arise.

LEMMA 3. *For $\beta > 0$, there is no equilibrium in which any bank offers a pooling contract.*

Proof. See Lemma C.2 in Appendix C. □

This is an interesting result as, although pooling would be the unique equilibrium for a monopsonistic bank ($\beta = 0$)[5], competition rules out the existence of pooling equilibria. In order to see this, suppose first the bad bank offers a pooling contract. The only candidate for this is a contract with a fixed wage of $F^B = \alpha\theta_H^B + (1-\alpha)\,\theta_L^B \equiv \overline{\theta}^B$ (and $w = 0$): Any $F^B > \overline{\theta}^B$ is weakly dominated by $F^B = \overline{\theta}^B$, and for any $F^B < \overline{\theta}^B$, the bad bank would have a profitable deviation to the good bank's best response.

Next, the good bank's unique best response to pooling by the bad bank is to offer a pooling contract as well: As the two types' reservation utilities are identical when the bad bank pools, there is no information rent that can be reduced via separating contracts. Thus, the good bank's unique best response to pooling is to offer one contract, with $F^G = F^B = \overline{\theta}^B$. However, this is no equilibrium as the bad bank then has a profitable deviation: It can offer a contract with $w > 0$ that is only attractive to the high type, and as the high type's expected output is greater than $\overline{\theta}^B$, the bad bank would earn positive profits.

Suppose next that the bad bank offers separating contracts while the good bank plays pooling. To see why this cannot be an equilibrium either, note that the good bank's unique best pooling response is a contract with $F^G = \widehat{U}_H$ as *(PCH)* is binding. But then the bad bank has the following profitable deviation: By increasing F_L^B, it can reduce the low type's imitation incentive. This lower imitation incentive allows to reduce w. Lowering w increases the high type's expected output, and the bad bank can hence offer more than $\widehat{U}_H$ to the high type, and can still earn positive profits.

We now turn to separating equilibria and summarize important insights in the following Lemma.

LEMMA 4. *In equilibrium, the following properties are satisfied: (i) (PCL) is binding; (ii) the bad bank offers the high type exactly her total expected output; (iii) the bad bank offers the low type a fixed wage that weakly exceeds her output.*

Proof. See Lemmas C.1 and C.2 in Appendix C. □

Recall first that we have distinguished two regions in the good bank's best response function in Proposition 1: Region 1 where *(PCL)* is non-binding, and region 2 where it is binding. *Part (i)* of the Lemma implies that, even though region 1 is part of the good bank's best response function, it cannot constitute an equilibrium as the *bad* bank always has a profitable deviation when *(PCL)* is non-binding. The reason is as follows: When *(PCL)* is non-binding, that is, when $F_L^G > F_L^B$, the bad bank can increase F_L^B without attracting the low type. Increasing F_L^B reduces the low type's imitation incentive, and hence allows to increase the high type's expected output by decreasing w. When the output increases, it follows that the bad bank can offer more to the high

[5] For $\beta = 0$, the monopsonistic good bank offers fixed wages of $F_H = F_L = 0$, the agents choose $\gamma_i = 0$, and consequently produce their maximum outputs θ_i.

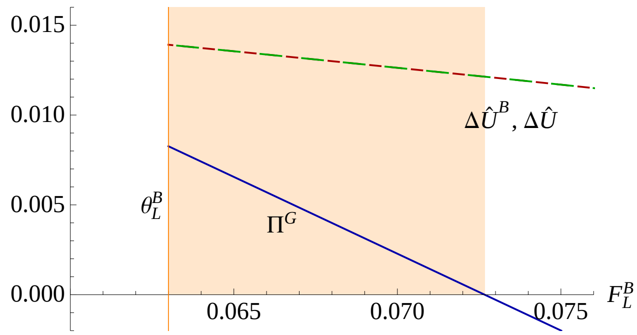

Figure 4: Illustration of the range of equilibria. Dashed line: $\Delta\widehat{U}^B$, wide dashed line: $\Delta\widehat{U}$, Π^G: good bank's profit. The range of equilibria is given by the shaded area. Here, $\overline{F}_L^B$ is given by the condition $\Pi^G \geq 0$, while $\underline{F}_L^B$ is bounded by θ_L^B. That *(PCL)* is binding for all F_L^B follows easily from $\Delta\widehat{U}^B = \Delta\widehat{U}$ and the binding *(PCH)*.

type and earn positive profits from her at the same time. Hence, any equilibrium is in region 2, where (PCL) is binding.

Part (ii) of the Lemma follows immediately from the fact that *(PCH)* is binding: As the good bank's offer to the high type only just matches the bad bank's offer, the bad bank would always have a profitable deviation when offering less than total expected output. And as offering more is weakly dominated, it must offer the high type exactly her total output.

Part (iii) states that the bad bank may well offer *more* than total output to the low type. That it would face losses when attracting her is irrelevant as long as the good bank matches the offer. Note carefully that offering the *low* type more than her output is *not* weakly dominated for the following reason: The higher F_L^B, the lower is the low type's imitation incentive, and the lower hence the bonus w required to avoid imitation. Hence, offering more to the low type than her output improves the offer the bad bank can make to the high type without violating the low type's incentive compatibility constraint.

Making use of Lemma 4, the equilibrium configuration is now best described in terms of the fixed wage the bad bank offers to the low type, F_L^B.[6] Let us denote the minimum and maximum wages offered to the L-type agent that may constitute an equilibrium by $\underline{F}_L^B$ and $\overline{F}_L^B$, respectively. We already know that $\underline{F}_L^B \geq \theta_L^B$ as otherwise the bad bank has a profitable deviation.

[6]Clearly, any change in F_L^B has also an impact on the other contract terms since we consider a setting where each bank offers two contracts, but the intuition can nicely be explained by referring to F_L^B only. Still, whenever we talk about changes in F_L^B, we need to keep in mind that all other contract terms are adjusted accordingly to fulfill the relevant (ICC)- and (PC)-constraints as well as the fact that the bad bank must earn zero profits.

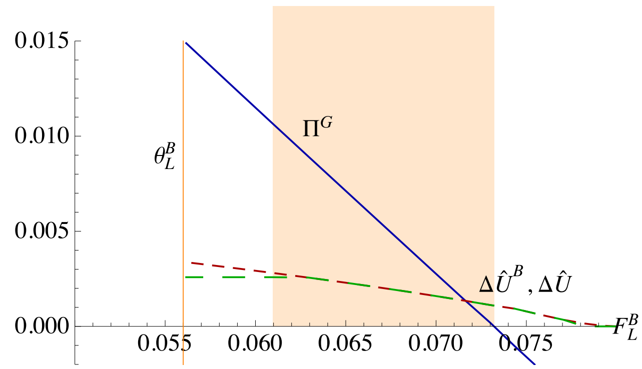

Figure 5: Illustration of the range of equilibria. Dashed line: $\Delta\widehat{U}^B$, wide dashed line: $\Delta\widehat{U}$, Π^G: good bank's profit. The range of equilibria is given by the shaded area. $\overline{F}_L^B$ is given by $\Pi^G \geq 0$, while $\underline{F}_L^B$ is bounded by the binding *(PCL)*.

For the interpretation of $\overline{F}_L^B$, just note that the good bank's profit is decreasing in F_L^B. For our example, this is illustrated in Figure iv by the line denoted Π^G, which shows the good bank's expected profit as a function of F_L^B. As the good bank will participate only if it expects non-negative profits, the upper bound $\overline{F}_L^B$ is given by $\Pi^G = 0$.

For the parameter values in Table i, we have $\underline{F}_L^B = \theta_L^B$, and that *(PCL)* is binding follows easily from observing that $\Delta\widehat{U}^B = \Delta\widehat{U}$ and the fact that *(PCH)* is binding. The interval $[\underline{F}_L^B, \overline{F}_L^B]$ is then given by the shaded area. Furthermore, we obtain the following figures in the good bank for the equilibrium associated with $\underline{F}_L^B$: The bonus fraction offered is $w^\star = 0.57$, the high type's portfolio choice is $\gamma_H = 0.57$, while the low type chooses $\gamma_L = 0$. The expected net gain from the high type is $\theta_H + \gamma_H (\xi_H - \theta_H) = 0.11$, which is below the return of the safe project of $\theta_H = 0.15$. Nevertheless, the expected net gain from the high type is still above the net gain from the low type given by $\theta_L = 0.07$ (this is illustrated in Figure i), as screening would otherwise be useless. Taking the effort costs of $C(\gamma_H) = 0.018$ into account, the overall loss in social welfare from the high type amounts to 39%.

Even though for the parameter values of our example $\underline{F}_L^B = \theta_L^B$ holds, an equilibrium may often require that the bad bank offers the low type strictly *more* than her total output, that is, $F_L^B > \theta_L^B$. This is the case whenever for $F_L^B = \theta_L^B$ the *(PCL)* is non-binding in the good bank's best response function. This case is illustrated in Figure v.[7] From Lemma 4 we know that the region where *(PCL)* is non-binding cannot constitute an equilibrium as the bad bank can always profitably deviate if the good bank offers the low type more than F_L^B. This establishes a lower bound $\underline{F}_L^B > \theta_L^B$.[8]

Summing up, in equilibrium there is a *lower* bound on F_L^B, denoted by $\underline{F}_L^B$, given

[7]The parameters for this example are available from the authors upon request.

[8]There is a second case where $\underline{F}_L^B > \theta_L^B$; here the bad bank itself has a profitable deviation if it offers the low type just her output. This arises if the bad bank can offer both types a higher wage by

by the requirement that *(PCL)* is binding in the good bank's best response and there is no profitable deviation for the bad bank. At the same time, there is an *upper* bound, denoted by $\overline{F}_L^B$, due to the good bank's participation constraint. This has two implications: If $\overline{F}_L^B < \underline{F}_L^B$, there exists no equilibrium – any F_L^B sufficiently high to ensure that *(PCL)* is binding would yield negative profits for the good bank, and any F_L^B that would yield non-negative profits would lead to a non-binding *(PCL)*, and hence to a profitable deviation for the bad bank. On the other hand, it is straightforward that whenever $\overline{F}_L^B > \underline{F}_L^B$, then multiple equilibria exist, all characterized by $F_L^{B,*} \in [\underline{F}_L^B, \overline{F}_L^B)$.

Arguing with F_L^B nicely captures some features of the equilibria, but it is, of course, an endogenous variable. Hence, the next step is to characterize the conditions for the existence of an equilibrium and its properties with respect to the exogenous variable β, the degree of competition for workers. In order to analyze the impact of β, we need to be consistent in equilibrium selection, since there may be multiple equilibria and since the bonus depends on the equilibrium chosen. Without loss of generality, we introduce the following Assumption:

ASSUMPTION 2. *For any β, we assume that the equilibrium with $F_L^B = \underline{F}_L^B$ is chosen.*

Let us first characterize the properties of the selected equilibrium, assuming it exists. Next, we prove existence.

PROPOSITION 2. *(i) The difference in the expected utilities the two agent types are offered by the bad bank, $\Delta\widehat{U}^B$, is (weakly) increasing in the degree of competition for agents, that is, $\frac{\partial}{\partial\beta}\Delta\widehat{U}^B \geq 0$. (ii) The bonus offered by the good bank is (weakly) increasing in β, that is, $\frac{\partial}{\partial\beta}w^*(\beta) \geq 0$. (iii) Social welfare is (weakly) decreasing in β.*

Proof. See Appendix C. □

The reason why $\Delta\widehat{U}^B$ is non-decreasing in β is straightforward: The lower is β, the lower is the productivity difference of the two types in the bad bank, and the lower is ceteris paribus also the difference in their expected utilities in the contracts offered by the bad bank. To see this, just assume that β converges to zero. Then, the utility offered to the high type, and hence also the difference in utilities, converges to zero as well.

For *part (ii)*, just recall that *(PCL)* is binding in equilibrium. But then, we already know from Proposition 1 in Section 4 that the bonus w offered to the high type is increasing in $\Delta\widehat{U}^B$. Thus, the problem of excessive risk-taking induced by bonuses becomes more pronounced when competition for workers increases. Of course, social

increasing the fixed wage for the low type, thus attracting both types, and making a profit at the same time (of course, in this deviation the high type will obviously not receive her expected output).

welfare in our model is strictly decreasing in γ (and hence in w) as this reduces expected returns and increases effort costs $C(\gamma)$ at the same time. Thus, *part (iii)* expresses that, in our model, competition for workers is detrimental to social welfare.

It remains to analyze the existence of equilibria. We find:

PROPOSITION 3. *There is some $0 < \overline{\beta} \leq 1$ such that an equilibrium in pure strategies exists for all $\beta \leq \overline{\beta}$.*

Proof. See Appendix C. □

Proposition 3 follows directly from three of our former results: From the last Proposition, we know that $\Delta\widehat{U}^B$ is weakly increasing in β. From Proposition 1 on the good bank's best response function, we know that *(PCL)* is binding if and only if $\Delta\widehat{U}^B$ is sufficiently small. And from Lemma 4, we know that there is no equilibrium if *(PCL)* is non-binding in the good bank's best response function. Hence, a necessary and sufficient condition for the existence of a pure- strategy equilibrium is that the good bank has sufficient labour market power.

6 Robustness

In our model, we restrict attention to non-negative bonuses (implying limited liability) that are linear in the excess return greater than θ_H. In Sections 6.1 and 6.2 we discuss the robustness of our results with respect to these two assumptions. All proofs and examples are provided in an Online-Appendix.[9] In Section 6.3, we discuss further assumptions of our model.

6.1 LIMITED LIABILITY

A natural way of relaxing the limited liability-assumption is to assume that the agent has some wealth V that can be deployed as a co-investment. To stick as closely as possible to our main model, assume that, in the contract designed for the high type, the co-investment is called upon whenever the portfolio return R is below θ_H. Define the percentage participation rate as τ, so that the agent is obliged to make a payment to the bank of the following form:

$$\text{payment} = \begin{cases} 0 & \text{if} \quad \tau(\theta_H - R) \leq 0 \\ \tau(\theta_H - R) & \text{if} \quad \tau(\theta_H - R) \in (0, V) \\ V & \text{if} \quad \tau(\theta_H - R) \geq V \ . \end{cases}$$

As the analysis in our general model is quite involved at no extra benefit, we restrict the analysis to log-returns that are normally distributed and to quadratic effort costs with $C(\gamma) = \frac{1}{2}c\gamma^2$. We then get the following result:

[9] Available at `http://www.revfin.org`.

PROPOSITION 4. *Suppose the agent has wealth V that can be taken as a co-investment. Then, there exists $\overline{V}$ such that the following results hold:* (i) *For $V \geq \overline{V}$, the good bank implements a first best, that is, $w = 0$.* (ii) *For $V < \overline{V}$, the participation rate in the contract designed for the high type is $\tau = 1$. The bonus is decreasing in V, and social welfare is increasing in V.*

Proof. See Online-Appendix □

According to the first part of Proposition 4 the problem of excessive risk-taking disappears if the agents's wealth that can be taken as a co-investment is sufficiently large. The reason is that the good bank can then offer the following two contracts: One contract designed for the low type that consists only of a fixed wage F_L, and one contract designed for the high type where the high type bears a sufficiently high fraction of the risk so that she prefers to choose $\gamma = 0$ as well. The contracts chosen by the two types are different even though both types choose $\gamma = 0$ in equilibrium. This allows to compensate both types according to their exit options $\widehat{U}_H^B$ and $\widehat{U}_L^B$.

The second part of the Proposition states that, for lower levels of deployable wealth V, relaxing the limited-liability constraint reduces but does not resolve the problem of positive bonuses and excessive risk-taking. The good bank mitigates the incentive problem by holding the H-type agent liable for any return below the safe return θ_H as long as she has sufficient wealth.

To see why demanding a participation rate of $\tau = 1$ is optimal for the bank, note first that the expected loss over all returns below θ_H is higher for the L-type agent due to the first order stochastic dominance property of the H-type's return function. Hence, the *difference* in the expected co-investment payments between the L-type and the H-type is increasing in τ. Even though the formal proof is challenging, it is straightforward that $\tau = 1$ is optimal: Consider a contract offer to the high type where, for $\tau < 1$ the low type would only just imitate. Now assume that the good bank increases both the participation rate τ and the fixed wage F_H^G such that the high type's expected utility remains the same. But then, the low type's utility from imitating decreases as she benefits from higher F_H^G just as the high type does, but her expected punishment from increasing τ is higher. Hence, for any expected utility of the H-type agent in the good bank given, a higher participation rate τ reduces the low type's imitation incentive in the same way as a higher bonus does: Just as the low type benefits less from higher bonuses, she is harmed more by a higher participation rate.

Finally, recall that $\gamma(w)$, the risk taken by the high type that the good bank implements in a separating equilibrium, depends on the low type's imitation incentive. And as the imitation incentive is decreasing in the participation rate and thus in the agent's wealth V, so is $\gamma(w)$. In our main model, the only device for reducing the low type's imitation incentive is to increase the bonus w. But with a co-investment instrument, the bank has now two devices, w and τ. And as both variables reduce the low type's imitation incentive for any expected utility of the high type given, the good bank can now implement a lower risk γ, and still prevent imitation by the bad type. This implies that social welfare is higher when the agent's wealth V increases.

6.2 BONUS LINEARITY

In our model, we restrict attention to bonuses that are linear in excess returns relative to a given threshold. Naturally, the question arises if the results are qualitatively robust when allowing for non-linear payment schemes. In the following, we wish to make two points in this respect: First, we show that whether non-linear bonus schemes can mitigate or even solve the problem of excessive risk-taking in our type of model depends on the information that the observed return provides on the agent's type and on the risk taken by the agent. Second, we briefly discuss why linear bonus schemes are predominantly observed in practice, and why restricting attention to linear bonuses is reasonable for our purposes even when more complex bonus schemes would, in principle, allow to reduce the problem of excessive risk-taking.

Regarding the informativeness of the return structure, we distinguish three cases:

Case 1: Sufficient information. In the first extreme case, the return is so informative as to the risk chosen that the bank could perfectly eliminate the problem of excessive risk-taking by paying bonuses only for specific return realizations. A sufficient condition for the existence of a first-best non-linear bonus scheme is that the safe project is deterministic and yields different returns for the two agent types. This is in fact the case in our model. As the safe project yields a certain return of θ_i^k, the bank can perfectly infer whether the agent has only invested in the safe project or not if the probability to realize θ_i^k is zero for $\gamma > 0$.

With such an information structure, the good bank can implement a first best separating equilibrium without information rent by offering two simple payment schemes: A first contract, designed for the high type, where the total payment is $W_H^G = \widehat{U}_H^B$ if and only if the return is θ_H^G, and zero otherwise; and a second contract designed for the low type where $W_L^G = \widehat{U}_L^B$ if the return is θ_L^G, and zero otherwise. Then, the best the two types can do is to accept the contracts designed for them, and to choose $\gamma = 0$. At the end of this section, we discuss why we have chosen such a simple and highly informative return structure, while nevertheless restricting attention to linear bonus schemes.

Case 2: No information. In the opposite case, the probability distribution over all *positive* returns provides no information at all on the risk actually taken or on the agent's type. In the Online-Appendix, we provide an example for such a situation and show that optimality of linear bonuses requires that the *ratio* of the probabilities for all positive returns be independent of the risk taken and be equal for both types. Still, the two types differ in the example as the overall probability for positive returns is higher for the high type. In such a situation, the bank can neither reduce the imitation incentive nor the incentive for excessive risk-taking by shifting bonus from one return to the other as the agent is only interested in the expected bonus over all positive returns.

Case 3: Intermediate information. In the Online-Appendix, we also provide an intermediate example where neither project offers a deterministic return, but where the probability ratios over positive returns differ for the two agent types. Then, the problem of excessive risk-taking can be mitigated by paying bonuses only for those returns that are relatively more likely for the high type. To see this, suppose there are only two positive returns, R_1 and R_2, and that the probability ratio $\frac{p(R_1)}{p(R_2)}$ is greater for the high type. If the bank shifts bonus from R_2 to R_1 such that the expected overall bonus payment remains the same for the high type, then the low type's imitation incentive decreases as she puts more weight on R_2, compared to the high type. Whether non-linear bonus schemes can then mitigate or even eliminate the problem of excessive risk-taking depends on whether the difference in the probability ratios is sufficiently high. We show in the Online-Appendix that, for these intermediate cases, all of our results remain valid even when the best available non-linear bonus scheme is implemented.

Summing up, we find that linear bonuses are optimal only in the extreme case where the return structure provides no information at all on the agent's type and choices. Nevertheless, the problem identified in our paper exists for intermediate cases as well. Tailoring the model to ensure that linear bonuses are optimal (case 2 above) requires the assumption of a very specific, and in our view unrealistic return structure. In reality, return structures typically provide at least some information on the risk chosen or on the agent's type that can be exploited by a refined bonus scheme. However, linear bonuses are predominantly used in practice (Holmström and Milgrom, 1987; Schmalensee, 1989; Milgrom and Roberts, 1992; Bhattacharya and Lafontaine, 1995), and we believe that this is not because return structures are completely uninformative, but rather because non-continuous bonus schemes such as in cases 1 and 3 are very fragile with regard to uncertainties in the return distributions. To see this, consider again case 1 and suppose that the bank assumes a safe project return of θ_H^G while the agent assumes $\theta_H^G + \varepsilon$. Then, she will not sign such a contract as, according to her belief, the bonus would be zero. Thus, the information assumptions required to eliminate the problem of excessive risk-taking with non-linear bonus schemes are very strong. Linear bonuses, in contrast, are more robust to varying beliefs on the exact return structure. Hence, for the points we make in this paper, restricting attention to linear bonuses is reasonable.

6.3 OTHER ASSUMPTIONS

Project risk. In our model, the risky project yields a lower expected return than the safe one. Although this may be deemed counterfactual, it allows to express the degree of excessive risk-taking in a simple way solely via γ, the percentage of total resources invested into the risky project. This point of view is shared by both the bank and society. We have considered two more realistic alternatives, both yielding qualitatively similar results, albeit at the expense of a significantly more involved model structure.

The first and in our view next best alternative is to assume expected returns of the risky project to be decreasing in γ, starting from above the safe project's return. Then, a cutoff-level $\widetilde{\gamma}$ can be derived that maximizes the bank's expected profit. Bonuses in a separating equilibrium will induce the high type to invest a larger percentage than $\widetilde{\gamma}$ in the risky project – a qualitatively similar result as in our model. The second alternative assumes that the risky project has consistently higher expected return than the safe project. This requires further assumptions to ensure that the bank prefers the safe project even with a lower return. Given the limited liability of banks and the bail out perspective, this would lead to questions that are clearly beyond our paper's purpose.

Competition for workers. While many papers analyze the impact of the market structure (competition) on the risk taken in the banking sector, our model is among the few who examine the impact of competition *for employees*. Although we consider a specific setting where the degree of labour market competition is captured by the simple parameter β, we feel safe in arguing that this accounts for the important features of competition for workers: High ability workers can only demand high wages if they can credibly threaten to switch employers; otherwise they are not able to benefit from their high capabilities as they have no negotiation power. Hence, β conveniently expresses the competitive threat bank B puts on bank G, and seems suitable for investigating if competition for workers is likely to induce bonuses leading to excessive risk taking. Moreover, the result of inefficient separating equilibria should hold in any model where the exit option of the more productive worker is higher.

Positive impacts of bonuses. In our model, social welfare is strictly decreasing in bonuses, which have only two effects: They allow to reduce the low type's information rent and they lead to excessive risk. As the first feature is purely re-distributive, the impact on social welfare is straightforward. Of course, we do not deny that there are also positive effects of bonuses, and our paper should be seen as complementary to that string of literature. These positive effects include higher effort incentives, higher incentives to actually invest in talent, and the possibility that unproductive workers are not employed at all because high-powered incentive schemes violate their participation constraint. Notwithstanding the importance of these well-known effects, our result that bonuses are *excessively* high should be qualitatively independent of whether these effects are taken into account or not.

One-period game. Although our game-theoretic model considers several stages with different information sets, the dynamic perspective is clearly limited. While we relate our findings to dynamic models in the literature review (Acharya et al., 2011; Pfeil and Inderst, 2010), some remarks on the potential robustness of our results vis-à-vis dynamic aspects are in order here.

A first question is concerned with the relationship of our model to the discussion of deferred bonuses. The main purpose of deferred bonuses is to mitigate the problem of "fake alpha", that is, the creation of excess short-term returns at the expense of high risks that materialize only after bonuses have been paid (Rajan, 2008). Postponing bonus payments most likely increases information and, hence, efficiency. A similar reasoning holds for bonus-malus-schemes or bonus-clawbacks where bonuses are either held in escrow and paid out only after risk-adjusted value creation has been ascertained after a longer period of time or are clawed back if value destruction has been discovered (Clementi et al., 2009). In our model, in contrast, there is only one return realization stage. The final return is hence equivalent to the risk-adjusted return, so that there is no conceptual difference between early and deferred bonuses.

Second, one might wish to consider the impact of a dynamic model where banks learn the agents' types from observing returns over time and make use of this information by re-negotiating the contract terms. This is related to the concept of ex-post settling up by Fama (1980), who discusses how learning about an agent's attributes may increase the intertemporal efficiency of labour contracts. Analyzing this aspect in depth would require a far-reaching model that is beyond the purpose of this paper, but some tentative remarks are possible: As in any separating equilibrium, the principal can infer the true agent type directly after the contract has been signed. As a consequence, he has an incentive to renegotiate the contract terms right away. However, as this would be anticipated by both agent types, screening will only be feasible with a renegotiation-proof device. Thus, our current model implicitly assumes that banks compete ex ante on contracts and cannot deviate ex post.[10] Still, in a multi-period model, one would expect that the bonus component offered to the high type would decrease over time as the low type's imitation incentive at the time the contract is signed is decreasing in the number of periods. However, a thorough discussion of these issues is left to future research as it requires a model where bonuses influence both the agent's effort and the choice of the project risk, so that positive and negative impacts of bonuses can hardly be decoupled.

7 Related Literature

Our paper contributes to the literature on executive compensation and competition in the market for managerial talent. Traditionally, the use of performance-related pay has been motivated by hidden action problems afflicting managers' work and by hidden information on managerial talent. In the literature on hidden information starting with Lazear (1986), it has been shown that highly productive workers self-select into pay-for-performance schemes, because they have stronger incentives to pay the higher

[10] We are grateful to an anonymous referee for pointing this out.

monitoring costs required by output-based pay or to bear the higher risk when output is uncertain (Moen and Rosen, 2005; Lazear, 2005; Balmaceda, 2009). Our paper abstains from monitoring costs and assumes risk-neutral agents, but screening is nevertheless feasible as the rate of substitution between fixed and variable payments is higher for high ability than for low ability agents.

In our model, the negative efficiency effect of variable payments is due to excessive risk-taking. This is in line with recent studies on negative incentive effects of performance-related pay that countervail the traditional view of bonuses as effort-increasing instruments (Holmström, 1979; Holmström and Milgrom, 1987; Gibbons, 1987; Baker, 1992). Bolton et al. (2010) demonstrate that, even though bank shareholders may have an interest in reducing managers' risk-taking incentives to lower the cost of debt, the unobservability of managers' actions will ultimately lead to risk shifting. To diminish these inefficiencies, the authors suggest tying bank managers' compensation not only to performance measures but also to measures of default risk (see also Bebchuk and Sperman (2010) and Edmans and Liu (2011)).[11]

While the papers in this vein recognize that compensation practices in highly-levered firms do not reflect the interests of all stakeholders, the modelling still relies on performance-related pay as an incentivizing instrument. In our paper, in contrast, we deliberately neglect the positive incentive effect in order to focus solely on the negative risk-taking problem. This allows us to conclude that, even despite the lack of any positive effort effects, banks do employ risk-enhancing performance-related payment schemes as screening devices.

The link between bonuses as screening instruments and efficiency-reducing risk-taking in our model is established via competition for managerial talent. The existence of a competitive labour market forces the principals to match the outside options of their managers. Labour market competition in this sense introduces an externality among firms that is similar to the externality effect of corporate governance observed by Hermalin and Weisbach (2006).

Acharya and Volpin (2010) and Dicks (2010) in this perspective consider the case where competition for managerial talent leads firms' choices of governance to be affected by the governance choices of their competitors. They show that firms may underinvest in governance as they do not internalize the benefit of their governance choice on competing firms. In our model, quite similarly, banks offer too high bonuses because they do not consider the negative effect that this has on the competing banks' bonus offerings. In Acharya and Volpin (2010), the externality causes too high performance pay combined with weak governance structures. This leads them to suggest bonus caps, among other governance-enhancing instruments, to increase efficiency.

Most closely related to our work are Acharya et al. (2011) who study the effect

[11] Hakenes and Schnabel (2010), in contrast, examine the optimal compensation scheme when bank managers exert insufficient effort and are prone to risk shifting. They show that the anticipation of public bail-outs, by reducing market discipline, lets a system of capped bonuses become the welfare optimal compensation scheme as it restricts excessive risk-taking while still exerting effort incentives.

of managerial competition on risk-taking in a dynamic model. In their model, risk-neutral banks compete for risk-averse managers who do not know their types initially. Managerial ability affects only risky projects while there is no effect on safe projects. As a consequence, high-ability managers should pick risky projects while low-ability managers should invest in safe projects. Project return materializes only after some time, so that banks need to learn the agents' types in order to efficiently allocate them to projects. Without competition for workers, even high types have no exit option. This allows banks to cross-subsidize low types at the expense of high types, and also reveals types over time. Assuming more projects than agents, however, firms will compete for workers, and equilibria arise in which types are never revealed and even low types invest in risky projects.

The main result in Acharya et al. (2011) is quite similar to our paper: Project choice is efficient without competition for workers, but poaching for talents leads to excessive risk-taking. Furthermore, as in our model, excessive risk-taking is induced by an externality problem between banks that arises only with competition. With respect to the underlying economic problems and their modelling, however, the two papers are complementary: In Acharya et al. (2011), there is no asymmetric information on types but uncertainty, that is, managers themselves initially do not know their types. Furthermore, if types are learned after one period, it is in the self-interest of bad (good) managers to invest in safe (risky) projects. Thus, the focus is on the impact of competition in a dynamic setting with uncertainty and learning on types. By contrast, our approach focuses on the impact of competition in a separating equilibrium with asymmetric information on types. Furthermore, competition is a continuous variable in our model which allows for comparative statics with respect to the competition parameter.

Thanassoulis (2011) also considers the effect of competition for banker talent on the optimal remuneration scheme. While our model focuses on excessive individual risk-taking by workers of unknown ability, his paper abstracts from information asymmetries, assumes that banks control the aggregate level of risk and emphasizes the risk sharing attribute of variable payments.

Learning about agents' capabilities is also a crucial factor in Inderst and Pfeil (2010). They examine whether deferred compensation may mitigate the problem of excessive risk-taking. Deferring compensation allows the bank to infer an agent's type, but also induces a cost, because agents have a higher time preference than banks. Inderst and Pfeil (2010) show that mandatory deferred compensation may lead to the implementation of projects with higher quality whenever the additional information is sufficiently valuable. As in our model, banks may benefit from bonus regulation, but for very different reasons: In Inderst and Pfeil (2010), mandatory deferred compensation may help to overcome the bank's commitment problem vis-à-vis the buyers of securities who benefit from higher quality. In our model, bonuses that are not in the bank's interest are not caused by commitment problems, but arise from competition for workers.

8 Conclusion

Current debates in the financial community suggest that bonuses are used to attract highly talented employees rather than to induce higher effort. At the same time, bank board members or compensation committees in charge of negotiating the remuneration packages seem to be well aware that high powered incentive schemes combined with limited liability are likely to trigger excessive risk-taking beyond the shareholders' and even the board members' interest.

We analyze whether competition for talents is likely to induce such excessive risk-taking as part of the equilibrium configuration. We consider a model where employees decide upon the fraction of resources to invest in a risky project that yields an expected return lower than that of a safe project. As the worker's ability is private information, bonuses are used as screening devices. In equilibrium, only high-ability workers receive bonuses, and excessive risk-taking is deliberately accepted in order to reduce low-ability workers' information rents. We show that bonuses are increasing in competition for bank employees, thereby confirming the impact of competitive pressures on the remuneration schemes as claimed by many industry representatives.

It turns out that excessive risk-taking does not only reduce social welfare but also the bank's own profits. Hence, our model suggests that legal restrictions on bonuses would increase profits and welfare at the same time. This assigns a generally positive role to regulatory interventions: Restricting bonus compensation may help to settle on a more efficient structure of fixed and performance-related payments by reducing excessive risk-taking. Of course, any regulatory restrictions on compensation schemes would have to account for a multitude of different factors, of which our paper emphasizes a combination that seems to have been neglected so far: Competition for talent and limited worker liability.

A Proofs of Section 3

We shall frequently make use of the following straightforward properties:

$$\frac{\partial}{\partial w} U_k(F,w) \geq 0, \qquad \text{for all } w \in [0,1], \tag{A.1}$$

$$U_H(F,w) > U_L(F,w), \qquad \text{for all } (F,w) \text{ with } w > 0. \tag{A.2}$$

The second property follows from the first-order stochastic dominance of ξ_H over ξ_L.

Proof of Lemma 1. The statement is valid for any agent type at any bank, so we omit the index denoting the bank type. Write the k-type agent's expected utility as

$$u^k(F,w,\gamma) = F + w\,\mathbb{E}\left[(\theta_k + \gamma(\xi_k - \theta_k) - \theta_H)^+\right] - C(\gamma).$$

If $\max_\gamma u^k(F,w,\gamma)$ has an interior solution, then, γ fulfills the following first- and second-order conditions of the expected utility with respect to γ:

$$u^k_\gamma(F,w,\gamma) := \frac{\partial}{\partial\gamma} u^k(F,w,\gamma) = w\,\mathbb{E}\left[(\xi_k - \theta_k)\,\mathbf{1}_{\left\{\xi_k - \theta_k \geq \frac{\theta_H - \theta_k}{\gamma}\right\}}\right] - C'(\gamma) = 0$$

$$u^k_{\gamma\gamma}(F,w,\gamma) := \frac{\partial^2}{\partial^2\gamma} u^k(F,w,\gamma) = \frac{w(\theta_H - \theta_k)^2\, f_k\left((\theta_H - \theta_k)/\gamma + \theta_k\right)}{\gamma^3} - C''(\gamma) \leq 0,$$

where f_k denotes the density of ξ_k.

(i) Suppose first that $\max_\gamma u^k(F,w,\gamma)$ has an interior solution. If $u^k_{\gamma\gamma}(F,w,\gamma) \neq 0$, then application of the Implicit Function Theorem yields

$$\gamma'(w) = -\frac{\frac{\partial}{\partial w} u^k_\gamma(F,w,\gamma)}{u^k_{\gamma\gamma}(F,w,\gamma)} = -\frac{\mathbb{E}\left[(\xi_k - \theta_k)\,\mathbf{1}_{\left\{\xi_k - \theta_k \geq \frac{\theta_H - \theta_k}{\gamma}\right\}}\right]}{\frac{w(\theta_H - \theta_k)^2\, f_k((\theta_H - \theta_k)/\gamma + \theta_k)}{\gamma^3} - C''(\gamma)},$$

which is easily seen to be strictly positive by the first- and second-order conditions above. If $u^k_{\gamma\gamma}(F,w,\gamma) = 0$, then $\gamma'(w) = 0$. Finally, if $\gamma(w) = 0$, then $\gamma'(w) \geq 0$, since $\gamma(0) = 0$.

(ii) It suffices to consider $F = 0$. Suppose first that $\gamma_k(w) \in (0,1)$. Then,

$$\begin{aligned}\frac{\partial}{\partial w} U_k(0,w) = {} & \mathbb{E}\left[(\theta_k + \gamma_k(w)(\xi_k - \theta_k) - \theta_H)^+\right] \\ & + \gamma_k'(w)\left\{ w\mathbb{E}\left[(\xi_k - \theta_k)\,\mathbf{1}_{\left\{\xi_k - \theta_k \geq \frac{\theta_H - \theta_k}{\gamma_k(w)}\right\}}\right] - C'(\gamma_k(w))\right\}.\end{aligned}$$

The term in braces is zero as a consequence of the first-order condition for $u^k_\gamma(w,F,\gamma)$. Hence, we must show that

$$\gamma_H(w)\mathbb{E}\left[(\xi_H - \theta_H)^+\right] > \mathbb{E}\left[(\theta_L + \gamma_L(w)(\xi_L - \theta_L) - \theta_H)^+\right], \quad w \in (0,1),$$

which can be re-written as

$$U_H(0,w)/w + C(\gamma_H(w)) > U_L(0,w)/w + C(\gamma_L(w)).$$

If $\gamma_H(w) \geq \gamma_L(w)$, then the claim follows directly from Inequality (9) and from $C' > 0$. Hence, it remains to show that $\gamma_H(w) \geq \gamma_L(w)$. This follows from $\mathbb{E}\left[(\xi_L - \theta_L)^+\right] \leq \mathbb{E}\left[(\xi_H - \theta_H)^+\right]$, since by the agents' first-order conditions,

$$\begin{aligned} w\,\mathbb{E}\left[(\xi_L - \theta_L)\,\mathbf{1}_{\left\{\xi_L - \theta_L \geq \frac{\theta_H - \theta_L}{\gamma_L(w)}\right\}}\right] &= C'(\gamma_L(w)) \\ &\leq w\,\mathbb{E}\left[(\xi_L - \theta_L)^+\right] \leq w\,\mathbb{E}\left[(\xi_H - \theta_H)^+\right] = C'(\gamma_H(w)). \end{aligned}$$

It remains to consider the case $\gamma_H(w) = 0$, which is trivial since $\gamma_H(w) = 0$ implies $\gamma_L(w) = 0$.

□

B Proofs of Section 4

Proof of Lemma 2. Throughout, denote the optimal contracts in a separating equilibrium by (F_H^*, w_H^*) and (F_L^*, w_L^*).

Let us first show that either *(ICCH)* or *(PCH)* is binding, formally $U_H(F_H^*, w_H^*) = \max(U_H(F_L^*, w_L^*), \widehat{U}_H^B)$. Re-write the bank's expected profit from meeting a high type (cf. first expectation of Equation (3)) as

$$\mathbb{E}\left[\gamma_H(w_H)(\xi_H - \theta_H)\right] + \theta_H - U_H(F_H, w_H) - C(\gamma_H(w_H)). \tag{B.1}$$

Choosing (F_H, w_H) such that $U_H(F_H, w_H)$ is strictly greater than $\max(U_H(F_L, w_L), \widehat{U}_H^B)$ cannot be optimal, since $U_H(F, w)$ is increasing in F and w, and since $\gamma_H(w)$ is increasing in w, and all terms involving $U_H(F_H, w_H)$ and $\gamma_H(w_H)$ in Equation (B.1) are negative. Thus the optimal choice such that *(ICCH)* and *(PCH)* are fulfilled is attained when one of the constraints is binding. It remains to check that the constraints *(ICCL)* and *(PCL)* are not violated. From Equation (A.2) we obtain

$$U_H(F_H^*, w_H^*) = \max(U_H(F_L^*, w_L^*), \widehat{U}_H^B) \geq U_L(F_L^*, w_L^*),$$

and

$$U_H(F_H^*, w_H^*) = \max(U_H(F_L^*, w_L^*), \widehat{U}_H^B) \geq \max(U_L(F_H^*, w_H^*), \widehat{U}_L^B),$$

which implies that none of the constraints is violated.

We now show that the claims of the Lemma hold in the order (i), (iii), (iv), (ii).

For (i), that is, $w_L^* = 0$, write the bank's expected profit from meeting a low type (cf. second expectation of (3)) as

$$\theta_L + \mathbb{E}\left[\gamma_L(w_L)\,(\xi_L - \theta_L)\right] - U_L(F_L, w_L) - C(\gamma_L(w_L)). \tag{B.2}$$

Set $w_L = 0$, which implies $\gamma_L(w_L) = 0$, and $U_L(F_L, 0) = F_L := \max(U_L(F_H^*, w_H^*), \widehat{U}_L^B)$. This maximises Equation (B.2) and satisfies the constraints *(ICCL)* and *(PCL)*. It remains to show that *(ICCH)* is fulfilled. Since $U_H(F_L, 0) = F_L$, *(ICCH)* may be re-written as $U_H(F_H^*, w_H^*) \geq F_L$. The claim now follows from Equation (A.2), the *(PCH)* constraint and

$$U_H(F_H^*, w_H^*) \geq \max(U_L(F_H^*, w_H^*), \widehat{U}_H^B) \geq \max(U_L(F_H^*, w_H^*), \widehat{U}_L^B) = F_L = U_H(F_L, 0).$$

Now consider (iii), that is, *(PCH)* is binding, formally, $U_H(F_H^*, w_H^*) = \widehat{U}_H^B$. Let (F_H, w_H) and F_L be a candidate that fulfills $U_H(F_H, w_H) = \max(F_L, \widehat{U}_H^B)$ (that is, *(PCH)* or *(ICCH)* binding), and suppose that $F_L > \widehat{U}_H^B$. We show that this is not optimal, that is, there exist $\overline{F}_H$, $\overline{w}_H$ and $\overline{F}_L$ such that $U_H(\overline{F}_H, \overline{w}_H) = \widehat{U}_H^B \geq \overline{F}_L$, with higher expected payoff to the bank than F_H, w_H, F_L and fulfilling the constraints. Set $\overline{w}_H := w_H$ and choose

$$\overline{F}_H = \widehat{U}_H^B - \mathbb{E}\left[w_H\, \gamma_H(w_H)\, (\xi_H - \theta_H)^+\right] + C(\gamma_H(w_H))\,.$$

Choose $\overline{F}_L = \max(U_L(\overline{F}_H, w_H), \widehat{U}_L^B)$; this choice of $\overline{F}_L$ fulfills the condition of the construction by Equation (A.2) and by virtue of $\widehat{U}_H^B \geq \widehat{U}_L^B$. Since

$$F_H = F_L - \mathbb{E}\left[w_H\, \gamma_H(w_H)\, (\xi_H - \theta_H)^+\right] + C(\gamma_H(w_H)),$$

we obtain for the expected payoff of the bank (cf. Equation (3)),

$$\begin{aligned}\alpha\, \mathbb{E}\Big[\gamma_H(w_H)\, (\xi_H - \theta_H) + \theta_H - \widehat{U}_H^B - C(\gamma_H(w_H))\Big] + (1-\alpha)\, \mathbb{E}\left[\theta_L - \overline{F}_L\right] \\ > \alpha\, \mathbb{E}\left[\gamma_H(w_H)\, (\xi_H - \theta_H) + \theta_H - F_L - C(\gamma_H(w_H))\right] + (1-\alpha)\, \mathbb{E}\left[\theta_L - F_L\right].\end{aligned}$$

It remains to observe that the constraints are fulfilled for $(\overline{F}_H, w_H)$ and $\overline{F}_L$: *(ICCH)* and *(PCH)* are fulfilled by construction, and *(ICCL)*and *(PCL)* are fulfilled by the choice of $\overline{F}_L$.

Next, consider (iv), that is *(ICCL)*is binding, formally $F_L^* = U_L(F_H^*, w_H^*)$. Using that *(PCH)* is binding, that is, $U_H(F_H^*, \widehat{w}_H) = \widehat{U}_H^B$, *(ICCL)* can be written as

$$U_H(F_H^*, w_H^*) - U_L(F_H^*, w_H^*) \geq \widehat{U}_H^B - F_L^*.$$

For the bank it is optimal to choose w_H as small as possible. Furthermore, Inequality (2) implies that $U_H(F_H, w_H) - U_L(F_H, w_H)$ is increasing in w_H. Hence, it is optimal to choose w_H such that *(ICCL)* is binding.

It remains to show property (ii), that is, if separating contracts are offered, then *(ICCH)* is non-binding. If *(ICCH)* is binding, then we obtain from the constraints and with the previous results that in the optimum

$$U_H(F_H, w_H) = U_H(F_L, w_L) \overset{w_L=0}{=} U_L(F_L, w_L) \overset{(PCH)\text{ binding}}{=} \widehat{U}_H^B \overset{(ICCL)\text{ binding}}{=} U_L(F_H, w_H),$$

which contradicts that separating contracts are offered. $\square$

Proof of Proposition 1. Observe first that w^* is nondecreasing in $\Delta\widehat{U}^B$ by property (2), as a higher $\Delta\widehat{U}^B$ relaxes the constraint and enlarges the set of feasible solutions.

Next, suppose that *(PCL)* is non-binding, that is, $U_H(0,w^*) - U_L(0,w^*) < \Delta\widehat{U}^B$. Then, w^* remains a (local) maximum as $\Delta\widehat{U}^B$ is increased. Since L has only one maximum, it follows that the maximum is global, so that w^* is constant for all $\Delta\widehat{U}^B > \Delta\widehat{U}^B_T$, with $\Delta\widehat{U}^B_T$ the infimum of $\Delta\widehat{U}^B$ such that (PCL) is non-binding. This characterizes region (1).

It follows that (PCL) is binding when $\Delta\widehat{U}^B \leq \Delta\widehat{U}^B_T$ and w^* is strictly increasing in $\Delta\widehat{U}^B$ as a consequence of property (2). □

C Proofs of Section 5

In this section, we formally derive the bad bank's best response function and provide the proofs of the statements found in Section 5. For efficiency reasons, the order of the statements and proofs deviates slightly from the order given in Section 5.

In general, a bank's best response function is as follows:

$$\max_{F_H,F_L,w_H,w_L} \alpha\mathbb{E}\left[\theta_H + \gamma_H(w_H)(\xi_H - \theta_H) - F_H - w_H\gamma_H(w_H)(\xi_H - \theta_H)^+\right]\mathbf{1}_{\{U_H(F_H,w_H)\geq\widehat{U}_H\}}$$
$$+(1-\alpha)\mathbb{E}\left[\theta_L + \gamma_L(w_L)(\xi_L - \theta_L) - F_L - w_L(\theta_L + \gamma_L(w_L)(\xi_L - \theta_L) - \theta_H)^+\right]\mathbf{1}_{\{U_L(F_L,w_L)\geq\widehat{U}_L\}},$$

subject to the constraints *(ICCL), (ICCH)*.[12]

Let us first derive some properties that hold in equilibrium.

LEMMA C.1. *In equilibrium, the bad bank's best response has – apart from the usual* (ICCL) *and* (ICCH) *constraints – the following properties:*

(i) the H-type agent receives her expected output;

(ii) the L-type agent receives at least her output;

(iii) the H-type's expected utility is maximal under the conditions stated.

Proof. (i) Recall that *(PCH)* is binding. If the bad bank offers less than total output, it can increase the offer to the high type, which will attract her and yields positive profits. To ensure that *(ICCL)* remains satisfied, the bad bank needs to increase w. This reduces the high type's expected output, but to a lower extent than the low type's utility from imitating. Hence, such a profitable deviation exists whenever the bad bank offers less than total output. Furthermore, offering the H-type more than her output is weakly dominated.

[12] Note that, in equilibrium, the good bank necessarily attracts both types, and the best response function then takes the usual form of Equation (3) subject to *(ICCH), (ICCL)* and additional constraints *(PCL), (PCH)*.

(ii) Suppose $F_L^B, F_L^G < \theta_L^B$. Then, the bad bank could profitably deviate by offering $F_L^B \in \left(F_L^G, \theta_L^B\right)$. Next, assume that $F_L^G > \theta_L^B$. Then, the bad bank has the following profitable deviation: It offers $F_L^B = F_L^G$ to the L-type who will hence still not be attracted, and this allows to increase F_H^B and to decrease w_H^B, while still fulfilling *(ICCL)*. This increases the high type's output, attracts her and yields positive profits.

(iii) Suppose the H-type's utility was not maximal. Then the bad bank could attract the H-type by maximizing her utility.

□

The bad bank's best response function thus takes the form[13]

$$\max_{F_L^B, F_H^B, w_H^B} U_H^B(F_H^B, w_H^B), \tag{C.1}$$

subject to the constraints

(i) $U_H^B(F_H^B, w_H^B) \geq F_L^B$ *(ICCH)*,

(ii) $F_L^B = U_L^B(F_H^B, w_H^B)$ *(ICCL)*,

(iii) $F_L^B \in [\theta_L^B, \widehat{U}_L]$ *(output L)*

(iv) $\mathbb{E}\left[\theta_H^B + \gamma_H^B(w_H^B)(\xi_H^B - \theta_H^B)\right]$
$= \mathbb{E}\left[F_H^B + w_H^B \gamma_H^B(w_H^B)(\xi_H^B - \theta_H^B)^+\right]$ *(output H)*.

In addition, we have the usual technical constraint $0 \leq w_H^B \leq 1$. Condition *(output L)* already reflects the fact that in equilibrium the good bank attracts both types, and since $F_L^B \geq \theta_L^B$, it follows that $\widehat{U}_L \geq \theta_L^B$.

As usual, $\gamma_k^B(w)$ denotes the fraction of the risky asset that maximizes a type k agent's expected utility, which in turn is given by

$$U_k^B(F, w) = \max_{\gamma \in [0,1]} F + w\mathbb{E}\left[\left(\theta_k^B + \gamma(\xi_k^B - \theta_k^B) - \theta_H^B\right)^+\right] - C^B(\gamma). \tag{C.2}$$

We can then examine properties that hold in equilibrium.

LEMMA C.2. *In equilibrium,*

(i) (PCL) *is binding;*

(ii) For $\beta > 0$, there is no equilibrium in which any bank offers a pooling contract.

[13] We state the problem in an already simplified form with $w_L^B = 0$ and *(ICCL)* binding. The proofs are similar to the respective proofs in the case of the good bank.

Proof. (i) Recall that in equilibrium *(PCH)* is binding. Suppose that *(PCL)* were not binding, that is $\widehat{F}_L^B < \widehat{U}_L$. The bad bank can then attract the H-type as follows: It offers the L-type agent a utility of $\widehat{U}_L$ – making *(PCL)* binding, but not attracting the L-type agent. In turn, it increases F_H^B and decreases w_H^B to match the binding *(ICCL)*. But this increases the H-type's output and utility, thus attracting the agent. But this cannot be an equilibrium as the good bank attracts both types in equilibrium.

(ii) We show first that the bad bank cannot offer a pooling contract. In equilibrium, the bad bank offers the H-type exactly her output, hence it would offer a pooling contract with wage θ_H^B, which yields negative expected utility

$$\Pi^B = \alpha\theta_H^B + (1-\alpha)\theta_L^B - \theta_H^B < 0.$$

Now suppose that the bad bank offers separating contracts and the good bank offers a pooling contract. But this contradicts the binding *(PCH)* and *(PCL)* conditions, and hence cannot be an equilibrium.

□

We state sufficient conditions for an equilibrium.

PROPOSITION C.1. *Let the good bank's and bad bank's best responses be as in Equations (3) and (C.1), respectively. A set of contracts is an equilibrium if and only if* (PCL) *is binding,* $F_L^B < \theta_H^B$ *and the good bank's expected payoff is non-negative.*

Proof. The "only if" part follows from the previous Lemma and from the exclusion of weakly dominated strategies. The "if" part: We must show that given the good bank's best response, the bad bank cannot unilaterally offer a set of contracts to increase its utility. As the good bank's expected utility is non-negative, it actually places its bid. As $F_L^B < \theta_H^B$, the bad bank's offer is not weakly dominated. Recall that the bad bank's best response is such that the L-type is offered at least her output and the H-type is offered exactly her output. If the bad bank increased its bid to either type, it would attract the type because of the binding *(PCH)* and *(PCL)* conditions. But both types are offered at least their output, so it cannot make a profit by offering more. □

Via the binding constraints *(ICCL)* and *(output H)* the problem can be simplified to one variable; furthermore, observe that *(ICCH)* is automatically fulfilled. First set:

$$F_H^B(w) := \mathbb{E}\left[\theta_H^B + \gamma_H^B(w)(\xi_H^B - \theta_H^B)\right] - \mathbb{E}\left[w\,\gamma_H^B(w)(\xi_H^B - \theta_H^B)^+\right] \tag{C.3}$$

$$F_L^B(w) := U_L^B(F_H^B(w), w). \tag{C.4}$$

Also, since in equilibrium *(PCL)* is binding, we have $F_L^B(w) = \widehat{U}_L$. Furthermore, $F_L^B(w)$ is strictly decreasing, as can be seen by observing that $U_H^B(F_H(w), w)$ is strictly decreasing by *(output H)* and because of property (2). The unique solution to the optimization problem is then given by w such that

$$F_L^B(w) = \widehat{U}_L.$$

The solution depends on $\widehat{U}_L$, the utility the L-type obtains from the good bank. In general, there is no unique equilibrium, and the range of equilibria depends on a number of factors outlined in the following.

To determine the minimum wage $\underline{F}_L^B$ that the bad bank offers, we first calculate the minimum $\widehat{U}_L$ where the bad bank's best response function takes the form of Equation (C.1). We call this candidate $\widehat{U}_{L,\min}$. The wage $\underline{F}_L^B$ is then determined as the minimum wage greater than $\widehat{U}_{L,\min}$ where *(PCL)* is binding. For the maximum wage $\overline{F}_L^B$ it must be ensured that the good bank's profit, Π^G, is non-negative. Further, $\overline{F}_L^B < \theta_H^B$ because of the exclusion of weakly dominated strategies. These reflections are also illustrated in Figures iv and v in the main paper.

We determine the minimum $\widehat{U}_L$.

LEMMA C.3. *The minimum $\widehat{U}_L$ that fulfills the bad bank's best response (C.1) is given by*

$$\widehat{U}_{L,\min} := \min_{F_H^B, w_H^B} U_L^B(F_H^B, w_H^B), \tag{C.5}$$

subject to (ICCL)*,* (ICCH) *and*

(i) $U_H^B(F_H^B, w_H^B) = \widehat{U}_{H,\min}$,

(ii) $\mathbb{E}\left[\theta_H^B + \gamma_H^B(w_H^B)(\xi_H^B - \theta_H^B)\right]$
$= F_H^B + w_H^B\gamma_H^B(w_H^B)\mathbb{E}\left[(\xi_H^B - \theta_H^B)^+\right]$ (output H)*,*

where $\widehat{U}_{H,\min}$ is given by

$$\widehat{U}_{H,\min} := \max_{F_L^B, F_H^B, w_H^B} U_H^B(F_H^B, w_H^B), \tag{C.6}$$

subject to (ICCL)*,* (ICCH) *and*

(i) $\alpha\mathbb{E}\left[\theta_H^B + \gamma_H^B(w_H^B)(\xi_H^B - \theta_H^B) - F_H^B - w_H^B\gamma_H^B(w_H^B)(\xi_H^B - \theta_H^B)^+\right]$
$+(1-\alpha)\mathbb{E}\left[\theta_L^B - F_L^B\right] = 0$ (break-even)*,*

(ii) $F_L^B \geq \theta_L^B$.

Let us first comment on the Lemma before providing the proof. Because of the break-even condition, $\widehat{U}_{H,\min}$ is the greatest utility the bad bank can offer to the H-type *regardless* of the actions of the good bank. However, since it may be optimal to offer the L-type more than her output, the H-type may be offered less than her output in optimization problem (C.6). Hence, this problem may fail to yield an equilibrium contract. However, any offer by the good bank smaller than $\widehat{U}_{H,\min}$ allows the bad bank to attract the H-type hence the minimum that the good bank must offer the H-type is $\widehat{U}_{H,\min}$. Finally, problem (C.5) finds the smallest utility to be offered to the L-type that fulfills the conditions for an equilibrium when offering $\widehat{U}_{H,\min}$ to the H-type. Observe further that $\widehat{U}_{L,\min} \geq \theta_L^B$, as $\widehat{U}_{H,\min}$ is greater than the utility offered to the H-type when both agents receive their respective outputs. Note that the solution to problem (C.5) is only an equilibrium candidate, but it may fail to be an equilibrium for two reasons: The good bank's best response may have a non-binding *(PCL),* and the good bank's expected utility may be negative.

Proof. Suppose that the bad bank offers $F_L^B < \widehat{U}_{L,\min}$. This implies for the utility U_H offered to the H-type that $U_H < \widehat{U}_{H,\min}$. Then, the bad bank can instead offer contracts with greater utilities to both agents, e.g. with slightly smaller utilities than implied by the optimization problem (C.6) involving the break-even condition and hence generate a positive expected utility. It remains to observe that $\widehat{U}_L$ is the smallest candidate that fulfills Equation (C.1). This is a consequence of $\widehat{U}_{H,\min}$ being the greatest utility offered to the H-type given $\widehat{U}_{L,\min}$ and by noting that all constraints of problem (C.1) are fulfilled. $\square$

We can now restate Proposition C.1 in terms of F_L^B.

PROPOSITION C.2. *An equilibrium exists if and only if* $F_L^B \in [\underline{F}_L^B, \overline{F}_L^B]$.

Proof. The "only if" part follows by definition of $\underline{F}_L^B$ and $\overline{F}_L^B$. For the "if" part, we show that the conditions of Proposition C.1 are fulfilled. We first show that *(PCL)* is binding for any $F_L^B \geq \underline{F}_L^B$. We have already derived that $F_L^B(w)$ is decreasing in w, so that conversely, w decreases as $F_L^B(w)$ increases. Re-write Equation (C.4) as $F_H^B(w) = U_L^B(0,w) - F_L^B(w)$, so that $\Delta\widehat{U}^B = F_L^B + U_H^B(0,w) - U_L^B(0,w) - F_L^B = U_H^B(0,w) - U_L^B(0,w)$, which decreases as w decreases (this is Lemma 1 (ii) applied to the bad bank). This implies that the good bank's best response remains in region 1 of Proposition 1 as F_L^B increases, hence *(PCL)* is binding. Further, by definition, $\overline{F}_L^B < \theta_H^B$. Finally, it remains to show that the good bank's expected utility is positive for $F_L^B < \overline{F}_L^B$. For this, consider the good bank's best response function as given by Equation (4). Clearly, if only $F_L^B = \widehat{U}_L^B$ were increasing and $\widehat{U}_H^B$ were to remain constant, then the good bank's best response L would be decreasing. Now, additionally increasing $\widehat{U}_H^B$ cannot increase L, as then a non-binding *(PCH)* condition would have been optimal before.

To complete the proof, it remains to observe that the best response functions of the good bank and the bad bank are indeed of the special forms Equation (3) and Equation (C.1), respectively, whenever $F_L^B \in [\underline{F}_L^B, \overline{F}_L^B]$. But this follows directly by observing that the good bank has a positive payoff only if it attracts both agents, while the bad bank does not attract any agent. □

Proof of Proposition 2. For the first statement, we show that both $\widehat{U}_H^B$ and $\widehat{U}_L^B$ are linear in β. Recall first that $\xi_H^B, \xi_L^B, \theta_H^B, \theta_L^B$ and C^B are linear in β by assumption. It follows directly from Equation (1) that $\gamma_k^B,\ k \in \{H, L\}$ does not depend on β. This implies that $F_H^B(w)$ and $F_L^B(w)$ as given by Equation (C.3) and Equation (C.4), respectively, are linear in β. Finally, it is straightforward that $U_H^B(F_H^B(w), w)$ is linear in β.

The second statement is a straightforward consequence of the first statement and the fact that the bonus w^* is strictly increasing in $\Delta\widehat{U}^B$ when *(PCL)* is binding (see region 1 of Proposition 1).

That social welfare is decreasing follows from the fact that the agents' output is decreasing in w. □

Proof of Proposition 3. It is sufficient to determine whether the equilibrium with $\underline{F}_L^B$ exists as this maximizes the good bank's expected utility Π^G among all equilibria. Write the latter as

$$\begin{aligned}\Pi^G = \alpha &\left\{\theta_H + \gamma_H(w^*)\mathbb{E}\left[\xi_H - \theta_H\right] - \underline{U}_H^B - C(\gamma_H(w^*))\right\} \\ &+ (1-\alpha)\left\{\theta_L - \underline{F}_L^B - C(0)\right\},\end{aligned}$$

where we have already used that *(PCL)* is binding, which holds for β small enough. It remains to show that for $\beta \leq \overline{\beta}$ the payoff Π^G is nonnegative, while for $\beta > \overline{\beta}$ it is negative.

The expected utility derived from the H-type agent is always non-negative due to the condition *(output H)* of the bad bank and the binding *(PCH)*. For β small enough we have $\theta_H^B \leq \theta_L$, and from $\underline{F}_L^B < \underline{U}_H^B < \theta_H^B$ it follows that $\Pi^G \geq 0$. Finally, $\dfrac{\partial}{\partial\beta}\Pi^G(\beta) \leq 0$, since $\underline{F}_L^B$, $\underline{U}_H^B$ and w^* are all nondecreasing in β. □

References

[1] Acharya, V. and Volpin, P. (2010) Corporate Governance Externalities, *Review of Finance* **14**, 1-33.

[2] Acharya, V., Pagano, M. and Volpin, P. (2011) Seeking Alpha: Excess Risk Taking and Competition for Managerial Talent, unpublished working paper, New York University.

[3] Baker, G. (1992) Incentive Contracts and Performance Measurement, *Journal of Political Economy* **100**, 598-614.

[4] Balachandran, S., Kogut, B. and Harnal, H. (2010) The Probability of Default, Excessive Risk, and Executive Compensation: A Study of Financial Services Firms from 1995-2008, unpublished working paper, Columbia Business School.

[5] Balmaceda, F. (2009) Uncertainty, Pay for Performance, and Asymmetric Information, *Journal of Law, Economics and Organization* **25**, 400-441.

[6] Bebchuk, L. A., Cohen, A. and Spamann, H. (2010) The Wages of Failure: Executive Compensation at Bear Stearns and Lehman 2000-2008, *Yale Journal on Regulation* **27**, 257-282.

[7] Bebchuk, L. A. and Spamann, H. (2010) Regulating Banker's Pay, *Georgetown Law Journal* **98**, 247-287.

[8] Berndt, A., Hollifield, B. and Sandas, P. (2010) The role of mortgage brokers in the subprime crisis, NBER Working Paper No. 16175.

[9] Bolton, P., Mehran, H. and Shapiro, J. (2010) Executive Compensation and Risk Taking, Federal Reserve Bank of New York, Staff Report No. 456.

[10] Cheng, I.-H., Hong, H. and Scheinkman, J.A. (2011) Yesterday's heroes: compensation and creative risk-taking, NBER Working Paper No. 16176.

[11] Clementi, G.L., Cooley, T.F., Richardson, M. and Walter, I. (2009) Rethinking Compensation in Financial Firms, in: V. Acharya and M. Richardson (eds.), *Restoring Financial Stability: How to Repair a Failed System*, John Wiley & Sons, pp. 189-214.

[12] DeYoung, R., Peng, E.Y. and Yan, M. (2010) Executive Compensation and Business Policy Choices at U.S. Commercial Banks, Research Working Paper 10-02, The Federal Reserve Bank of Kansas City.

[13] Dicks, D.L. (2010), Executive Compensation, Incentives, and the Role for Corporate Governance Regulation, unpublished working paper, University of North Carolina.

[14] Dittmann, I. and Yu, K.-C. (2009) How important are risk-taking incentives in executive compensation? Tinbergen Institute Discussion Paper 2009-076/2.

[15] Edmans, A. and Liu, Q. (2011) Inside Debt, *Review of Finance* **15**, 75-102.

[16] Fahlenbrach, R. and Stulz, R.M. (2011) Bank CEO Incentives and the Credit Crisis, *Journal of Financial Economics* **99**, 11-26.

[17] Fama, E.F. (1980) Agency Problems and the Theory of the Firm, *The Journal of Political Economy* **88**, 288-307.

[18] Flannery, M. (1998) Using Market Information in Prudential Banking Supervision: A Review of U.S. Evidence, *Journal of Money, Credit and Banking* **30**, 273-305.

[19] Gibbons, R. (1987) Piece-Rate Incentive Schemes, *Journal of Labour Economics* **5**, 413-429.

[20] Gropp, R., Hakenes, H. and Schnabel, I. (2011) Competition, Risk-Shifting, and Public Bail-out Policies, *Review of Financial Studies* **24**, 2084-2120.

[21] Hakenes, H. and Schnabel, I. (2010) Bank Bonuses and Bail-outs, unpublished working paper, University of Mainz.

[22] Hermalin, B. and Weisbach, M. (2006) A Framework for Assessing Corporate Governance Reform, unpublished working paper, UC Berkeley.

[23] Holmström, B. (1979) Moral Hazard and Observability, *The Bell Journal of Economics* **10**, 74-91.

[24] Holmstrom, B. and Milgrom, P. (1987) Aggregation and Linearity in the Provision of Intertemporal Incentives, *Econometrica* **55**, 303-328.

[25] Inderst, R. and Pfeil, S. (2010) Securitization and Compensation in Financial Institutions, unpublished working paper, Goethe-University Frankfurt.

[26] Kashyap, A.K., Rajan, R.G. and Stein, J.C. (2008) Rethinking Capital Regulation, in: Maintaining Stability in a Changing Financial System, Federal Reserve Bank of Kansas City, Kansas City, pp. 431-471.

[27] Lazear, E.P. (1986) Salaries and Piece Rates, *Journal of Business* **59**, 405-431.

[28] Lazear, E.P. (2005) Output-based Pay: Incentives or Sorting? *Research in Labour Economics* **23**, 1-25.

[29] Moen, E.R. and Rosen, A. (2005) Performance Pay and Adverse Selection, *Scandinavian Journal of Economics* **107**, 279-298.

[30] Murphy, K.J. (2009) Compensation Structure and Systemic Risk. USC Marshall School of Business, Marshall Research Paper Series, Working Paper FBE 34-09.

[31] Organisation for Economic Co-Operation and Development (2009) Corporate Governance and the Financial Crisis - Key Findings and Main Messages. OECD.

[32] Rajan, R. (2008) Bankers' pay is deeply flawed, Financial Times, January 8.

[33] Sironi, A. (2003) Testing for Market Discipline in the European Banking Industry: Evidence from Subordinated Debt Issues, *Journal of Money, Credit and Banking* **35**, 443-472.

[34] Stoughton, N.M. and Zechner, J. (2007) Optimal Capital Allocation Using RAROC and EVA, *Journal of Financial Intermediation* **16**, 312-342.

[35] Suntheim, F. (2010) Managerial Compensation in the Financial Service Industry, unpublished working paper, Bocconi University.

[36] Taleb, N. (2009) How bank bonuses let us all down, Financial Times, February 25.

[37] Tett, G. (2009) What bankers can learn from Chelsea football club, Financial Times, September 11.

[38] Thanassoulis, J. (2011) The case for intervening in bankers' pay, *Journal of Finance* forthcoming.

Online Appendix for
"Competition, bonuses, and risk-taking in the banking industry"

D Additional proofs

D.1 PROOFS OF SECTION 6.1

As outlined in Section 6.1, we assume that the agent has wealth V that can be taken as a co-investment. Assume that the good bank offers two contracts, a fixed wage contract designed for the low type, and a contract designed for the high type where the agent bears a fraction τ of all returns below θ_H as long as this does not exceed her wealth V. We show that it is optimal for the good bank to offer a contract with $\tau = 1$. Formally, the co-investment scheme prescribes the following payments from the agent to the bank:

$$\text{payment} = \begin{cases} 0 & \text{if} \quad \tau(\theta_H - R) \leq 0 \\ \tau(\theta_H - R) & \text{if} \quad \tau(\theta_H - R) \in (0, V) \\ V & \text{if} \quad \tau(\theta_H - R) \geq V \ . \end{cases}$$

It follows that a type-k agent's utility when choosing the contract designed for the high type, and when maximizing utility with respect to γ is

$$\begin{aligned} U_k(F, w, \tau, V) = \max_{\gamma \in [0,1]} & F + w\mathbb{E}\left[(\theta_k + \gamma(\xi_k - \theta_k) - \theta_H)^+\right] - C(\gamma) \\ & - \tau\mathbb{E}\left[(\theta_H - \theta_k + \gamma(\theta_k - \xi_k))\ \mathbf{1}_{\{\theta_H - \theta_k + \gamma(\theta_k - \xi_k) \in (0, V/\tau]\}}\right] \\ & - V\,\mathbb{P}\left(\xi_k \leq \theta_k + \frac{\theta_H - \theta_k - V/\tau}{\gamma}\right). \end{aligned}$$

The choice of the risky asset does now not only depend on the bonus w, but also on the wealth V and the participation rate τ. As usual, we denote a type-k agent's optimal choice by $\gamma_k(w, \tau, V)$.

The good bank's expected profit is given by (as before $w_L = 0$, *(PCH)* is binding):

$$\max_{w, F_L, \tau} \alpha \left\{\theta_H + \gamma_H(w, \tau, V)\mathbb{E}\left[\xi_H - \theta_H\right] - \widehat{U}_H^B - C(\gamma_H(w, \tau, V))\right\} + (1 - \alpha)\left\{\theta_L - F_L\right\},$$

subject to $F_L \geq U_L(F_H, w, \tau, V)$ *(ICCL)* and $F_L \geq \widehat{U}_L^B$ *(PCL)*, where the last condition is binding in equilibrium.

If *(ICCL)* is binding, then the good bank's expected profit is

$$\begin{aligned} \max_{w, \tau} \alpha & \left\{\theta_H + \gamma_H(w, \tau, V)\mathbb{E}\left[\xi_H - \theta_H\right] - \widehat{U}_H^B - C(\gamma_H(w, \tau, V))\right\} \\ & + (1 - \alpha)\left\{\theta_L - \widehat{U}_H^B + U_H(0, w, \tau, V) - U_L(0, w, \tau, V)\right\}, \quad \text{(D.1)} \end{aligned}$$

subject to *(PCL)*, which is binding in equilibrium, and given by

$$U_H(0, w, \tau, V) - U_L(0, w, \tau, V) = \Delta \widehat{U}^B.$$

As mentioned in the text, we now restrict attention to a more specific return function for the risky project. Specifically, we assume that the log-returns of the risky project are normally distributed, and that $(1+\theta_L)\,\mathbf{e}^{\mu_H - \mu_L} = 1+\theta_H$.[14] Furthermore, effort costs are given by $C = \dfrac{1}{2}c\gamma^2$, and to exclude technical problems, we assume that marginal costs are not too high, $c \leq \mathbb{E}\left[(\xi_H - \theta_H)^+\right]/2$.[15] To ease notation, we write γ_L and γ_H to denote $\gamma_L(w, \tau, V)$ and $\gamma_H(w, \tau, V)$, respectively.

As a prerequisite for proving Proposition 4, we need to show that the privately optimal risk taken by the high type is increasing in the bonus w, but decreasing in the wealth V and the participation rate τ. This is expressed in the following Lemma:

LEMMA D.1. *For the H-type agent we have*

$$\frac{\partial}{\partial w}\gamma_H(w, \tau, V) \geq 0 \tag{D.2}$$
$$\frac{\partial}{\partial \tau}\gamma_H(w, \tau, V) \leq 0 \tag{D.3}$$
$$\frac{\partial}{\partial V}\gamma_H(w, \tau, V) \leq 0. \tag{D.4}$$

Proof. All statements are proved with the Implicit Function Theorem. The H-type agent's expected utility satisfies the following first-order condition:

$$\frac{\partial}{\partial \gamma}u_H(\gamma, w, \tau, V) = w\,\mathbb{E}\left[(\xi_H - \theta_H)^+\right] - \tau\mathbb{E}\left[(\theta_H - \xi_H)\,\mathbf{1}_{\{\theta_H - \xi_k \in (0, V/(\gamma\tau)]\}}\right] - C'(\gamma) = 0.$$

The second derivative is given by

$$\frac{\partial^2}{\partial^2 \gamma}u_H(\gamma, w, \tau, V) = \frac{V^2}{\gamma^3 \tau} f\left(\theta_H - \frac{V}{\gamma\tau}\right) - C''(\gamma), \tag{D.5}$$

which is negative when u_H has a maximum at γ. The derivatives with respect to w, γ

[14] Recall that log-returns are normally distributed if

$$\xi_k = \mathbf{e}^{\mu_k - \frac{\sigma^2}{2} + \sigma X} - 1, \quad k \in \{H, L\},$$

where X is standard normally distributed. In other words, $1 + \xi_k$ follows a log-normal distribution with parameters $(\mu_k - 1/2\sigma^2, \sigma^2)$.

[15] This condition can often be weakened, but further case distinctions do not provide additional insights. Hence, we simply state a sufficient condition.

and V are given by

$$\frac{\partial^2}{\partial\gamma\partial w}u_H(w,\gamma,\tau,V) = \mathbb{E}\left[(\xi_H-\theta_H)^+\right] \geq 0$$
$$\frac{\partial^2}{\partial\gamma\partial\tau}u_H(w,\gamma,\tau,V) = \frac{V^2}{\gamma^2\tau^2}f\left(\theta_H-\frac{V}{\gamma\tau}\right) - \mathbb{E}\left[(\theta_H-\xi_H)\,\mathbf{1}_{\{\theta_H-\xi_H\in(0,V/(\gamma\tau)]\}}\right]$$
$$\frac{\partial^2}{\partial\gamma\partial V}u_H(w,\gamma,\tau,V) = -\frac{V}{\gamma^2\tau}f\left(\theta_H-\frac{V}{\gamma\tau}\right) \leq 0.$$

The claims with respect to w and V follow directly.

For the case of τ we have to show that $\frac{\partial}{\partial\gamma\partial\tau}u_H \leq 0$. From Equation (D.5), from the first-order condition, and since C is quadratic, we obtain

$$\begin{aligned}
&\frac{V^2}{\gamma^2\tau^2}f\left(\theta_H-\frac{V}{\gamma\tau}\right) - \mathbb{E}\left[(\theta_H-\xi_H)\,\mathbf{1}_{\{\theta_H-\xi_H\in(0,V/(\gamma\tau)]\}}\right] \\
&\qquad = \frac{\gamma}{\tau}\left(\frac{V^2}{\gamma^3\tau}f\left(\theta_H-\frac{V}{\gamma\tau}\right)\right) - \frac{1}{\tau}\left(w\,\mathbb{E}\left[(\xi_H-\theta_H)^+\right]-C'(\gamma)\right) \\
&\qquad \leq \frac{1}{\tau}\left(\gamma C''(\gamma) - w\,\mathbb{E}\left[(\xi_H-\theta_H)^+\right]+C'(\gamma)\right) \qquad \text{(D.6)} \\
&\qquad = \frac{2C'(\gamma)-w\,\mathbb{E}\left[(\xi_H-\theta_H)^+\right]}{\tau} < 0,
\end{aligned}$$

since $c < \mathbb{E}\left[(\xi_H-\theta_H)^+\right]/2$ by assumption. $\square$

Next, recall from the main model that the bonus w can be used as a screening device because the high type benefits more from the bonus than the low type. Similarly, the co-investment rate τ mitigates the imitation problem if and only if, in expectation, the low type pays more than the high type. This requires first-order stochastic dominance with respect to losses, and the following Lemma expresses that this property holds:[16]

LEMMA D.2. *In equilibrium, losses relative to θ_H of the H-type first-order stochastically dominate the respective losses of the L-type, that is, for all $y < 0$,*

$$\mathbf{P}\left(\theta_L+\gamma_L(\xi_L-\theta_L)-\theta_H \leq y\right) - \mathbf{P}\left(\gamma_H(\xi_H-\theta_H) \leq y\right) \geq 0,$$

and further there exists $\bar{y} \in [-1,0]$ such that for all $y \geq \bar{y}$ the inequality is strict.

[16] Recall that $\theta_k+\gamma_k(\xi_k-\theta_k)-\theta_H$ is just agent k's return minus the benchmark θ_H.

Proof. We write $y = -x$ for ease of notation, and re-write

$$\begin{aligned}
\mathbf{P}\left(\theta_L + \gamma_L(\xi_L - \theta_L) - \theta_H \le -x\right) - \mathbf{P}\left(\gamma_H(\xi_H - \theta_H) \le -x\right) \\
= \mathbf{P}\left(\xi_L \le \frac{\theta_H - \theta_L - x}{\gamma_L} + \theta_L\right) - \mathbf{P}\left(\xi_H \le \theta_H - \frac{x}{\gamma_H}\right) \\
= \mathrm{N}\left(\frac{\ln(1 + \theta_L + (\theta_H - \theta_L - x)/\gamma_L) - \mu_L + 1/2\sigma^2}{\sigma}\right) \\
- \mathrm{N}\left(\frac{\ln(1 + \theta_H - x/\gamma_H) - \mu_H + 1/2\sigma^2}{\sigma}\right),
\end{aligned}$$

for $\ln(\ldots) \in \mathbb{R}$.[17] This is positive if

$$\gamma_L > \frac{\theta_H - \theta_L - x}{(1 + \theta_H - x/\gamma_H)\,\mathbf{e}^{\mu_L - \mu_H} - (1 + \theta_L)} = \gamma_H \frac{\theta_H - \theta_L - x}{-x\,\mathbf{e}^{\mu_L - \mu_H}}, \quad x > 0.$$

It is easily seen that the claim is fulfilled if $x \le \theta_H - \theta_L$ and also if $x > \theta_H - \theta_L$ and $\gamma_L \ge \gamma_H$ (the latter follows from the stochastic dominance property of ξ_H over ξ_L). We must therefore consider the case $x > \theta_H - \theta_L$ and $\gamma_L < \gamma_H$.

Now choose

$$\bar{\gamma}_L = \gamma_H \frac{x - (\theta_H - \theta_L)}{x\,\mathbf{e}^{\mu_L - \mu_H}} = \gamma_H \mathbf{e}^{\mu_H - \mu_L}\left(1 - \frac{\theta_H - \theta_L}{x}\right),$$

which implies $\mathbf{P}\left(\theta_L + \bar{\gamma}_L(\xi_L - \theta_L) - \theta_H \le -x\right) - \mathbf{P}\left(\gamma_H(\xi_H - \theta_H) \le -x\right) = 0$ and let us show that $\gamma_L > \bar{\gamma}_L$ for suitably large x.

Denoting by $u_k(0, w, \gamma, x)$ the k-type agent's expected payoff when choosing γ, let us now consider

$$\begin{aligned}
\frac{\partial}{\partial \gamma} u_k(0, w, \gamma, x) = \mathbb{E}\left((\xi_k - \theta_k)\,\mathbf{1}_{\{\gamma(\xi_k - \theta_k) + \theta_k - \theta_H \in (-x, 0]\}}\right] \\
+ w\mathbb{E}\left[(\xi_k - \theta_k)\,\mathbf{1}_{\{\gamma(\xi_k - \theta_k) + \theta_k - \theta_H \ge 0\}}\right] - C'(\gamma), \quad k \in \{H, L\}, \quad \text{(D.7)}
\end{aligned}$$

and let $\bar{x}$ be such that the first-order condition $\frac{\partial}{\partial \gamma} u_L(0, w, \gamma_L, x) = 0$ is satisfied for all $x \ge \bar{x}$.

By choice of $\bar{\gamma}_L$ we have $\bar{\gamma}_L(\xi_L - \theta_L) + \theta_L - \theta_H \ge -x$ if and only if $\gamma_H(\xi_H - \theta_H) \ge -x$. Furthermore, by noting that $\mathbf{e}^{\mu_H - \mu_L}(\xi_L - \theta_L) \sim (\xi_H - \theta_H)$, we have $\bar{\gamma}_L(\xi_L - \theta_L) + \theta_L - \theta_H < 0$ if and only if $\mathbf{e}^{\mu_H - \mu_L}(\xi_L - \theta_L) < \frac{x(\theta_H - \theta_L)}{(x - (\theta_H - \theta_L))\gamma_H}$.

[17] If $\ln(\ldots) \notin R$, that is, if the argument of ln is negative, then the corresponding probability is zero.

Again, by noting that $\xi_L - \theta_L \sim \mathbf{e}^{\mu_L-\mu_H}(\xi_H - \theta_H)$, we obtain from Equation (D.7)

$$
\begin{aligned}
\frac{\partial}{\partial\gamma} u_L(0,w,\bar{\gamma},x) = \mathbf{e}^{\mu_L-\mu_H}\mathbb{E}\left[(\xi_H-\theta_H)\,\mathbf{1}_{\{\xi_H-\theta_H\in(-x/\gamma_H,0]\}}\right] \\
&+ \mathbf{e}^{\mu_L-\mu_H}\,w\,\mathbb{E}\left[(\xi_H-\theta_H)^+\right] \\
&+ (1-w)\mathbf{e}^{\mu_L-\mu_H}\mathbb{E}\left[(\xi_H-\theta_H)\,\mathbf{1}_{\left\{\xi_H-\theta_H\in\left(0,\frac{\theta_H-\theta_L}{x-(\theta_H-\theta_L)}\,\frac{x}{\gamma_H}\right]\right\}}\right] \\
&- c\,\gamma_H\mathbf{e}^{\mu_H-\mu_L}\left(1-\frac{\theta_H-\theta_L}{x}\right) \\
&= (1-w)\mathbf{e}^{\mu_L-\mu_H}\mathbb{E}\left[(\xi_H-\theta_H)\,\mathbf{1}_{\left\{\xi_H-\theta_H\in\left(0,\frac{\theta_H-\theta_L}{x-(\theta_H-\theta_L)}\,\frac{x}{\gamma_H}\right]\right\}}\right] \\
&- c\,\underbrace{\gamma_H\left(\mathbf{e}^{\mu_H-\mu_L}-\mathbf{e}^{\mu_H-\mu_L}\frac{\theta_H-\theta_L}{x}-\mathbf{e}^{\mu_L-\mu_H}\right)}_{(*)}.
\end{aligned}
$$

It remains to show that $(*) < 0$. Observe that $(*)$ is increasing in x and that $x \le \gamma_H(1+\theta_H)$ (since otherwise $\mathbf{P}(\xi_H \le \theta_H - x/\gamma_H) = 0$). Hence, inserting $x = \gamma_H(1+\theta_H)$ into $(*)$ yields

$$
\begin{aligned}
\gamma_H\frac{1+\theta_H}{1+\theta_L} - \frac{1+\theta_H}{1+\theta_L}\,\frac{1+\theta_H-(1+\theta_L)}{1+\theta_H} - \gamma_H\frac{1+\theta_L}{1+\theta_H} \\
&= \frac{\gamma_H(1+\theta_H)^2-\gamma_H(1+\theta_L)^2-(1+\theta_H)^2+(1+\theta_L)(1+\theta_H)}{(1+\theta_L)(1+\theta_H)} \\
&= \frac{(1+\theta_H)^2(\gamma_H-1)-(1+\theta_L)(\gamma_H(1+\theta_L)-(1+\theta_H))}{(1+\theta_L)(1+\theta_H)} \\
&< 0.
\end{aligned}
$$

Hence, we have shown that $\frac{\partial}{\partial\gamma}u(0,w,\bar{\gamma},x) > 0$, for all w, which implies that $\gamma_L > \bar{\gamma}_L$, for all x. For $x < \bar{x}$ the first-order condition does not hold, in which case $\gamma_L = 0$ (cf. Lemma D.1, Equation (D.3). □

From the last two Lemmas, we know that the risk taken by the high agent is decreasing in τ, and that the low type's imitation incentive is, for any expected utility of the high type, also decreasing in τ. In other words, τ has only positive impacts for the good bank, but no negative consequences. We can then prove Proposition 4.

Proof of Proposition 4. (i) First, *(ICCL)* will be non-binding if no bonus component is necessary to prevent the L-type from imitating, that is, if $\max_\tau U_H(0,0,\tau,V) - U_L(0,0,\tau,V) > \Delta\widehat{U}^B$. In this case the principal offers contracts $(F_H,0,\tau^*)$ and $(F_L,0,0)$, with $F_H = \widehat{U}^B_H - U_H(0,0,\tau,V)$, $F_L = \widehat{U}^B_L$ and τ^* maximizing

$U_H(0,0,\tau,V) - U_L(0,0,\tau,V)$, respectively. The L-type agent will not imitate since $U_L(F_H,0,\tau^*,V) = \widehat{U}_H^B - U_H(0,0,\tau^*,V) + U_L(0,0,\tau^*,V) < \widehat{U}_H^B - \Delta U^B = \widehat{U}_L^B$. We show below that $\frac{\partial}{\partial\tau} U_H(0,w,\tau,V) - U_L(0,w,\tau,V) \geq 0$, so that $\tau^* = 1$.

On the other hand, if *(ICCL)* is binding, then it is easily seen from Equation (D.1) that the good bank's expected profit is decreasing in $\gamma_H(w,\tau,V)$. Furthermore, $\frac{\partial}{\partial w}\gamma_H(w,\tau,V) \geq 0$ and $\frac{\partial}{\partial\tau}\gamma_H(w,\tau,V) \leq 0$ by Lemma D.1, so that decreasing w and increasing τ benefits the good bank. It remains to verify that this is consistent with the binding *(PCL)* condition.

Note first that in equilibrium necessarily $\frac{\partial}{\partial w}\left(U_H(0,w,\tau,V) - U_L(0,w,\tau,V)\right) \geq 0$, for if the derivative is negative, then the bank can increase its expected profit by decreasing w.

It remains to show that the difference in agents' utilities is nondecreasing in τ:

$$\begin{aligned}\frac{\partial}{\partial\tau} U_H(0,w,\tau,V) - U_L(0,w,\tau,V) &= \mathbb{E}\left[\gamma_H(\xi_H - \theta_H)\,\mathbf{1}_{\{\gamma_H(\xi_H-\theta_H)\in(-V/\tau,0]\}}\right] \\ &\quad - \mathbb{E}\left[\theta_L - \theta_H + \gamma_L(\xi_L - \theta_L)\,\mathbf{1}_{\{\theta_L-\theta_H+\gamma_L(\xi_L-\theta_L)\in(-V/\tau,0]\}}\right] \geq 0,\end{aligned}$$

where the last step follows from the first-order stochastic dominance property of losses, see Lemma D.2.

(ii) The argument is equivalent to part *(i)* and by noting that

$$\begin{aligned}&\frac{\partial}{\partial V} U_H(0,w,1,V) - U_L(0,w,1,V) \\ &\qquad = \mathbf{P}\left(\theta_L + \gamma_L(\xi_L - \theta_L) - \theta_H \leq -V\right) - \mathbf{P}\left(\gamma_H(\xi_H - \theta_H) \leq -V\right)\end{aligned}$$

is greater zero by Lemma D.2.

□

D.2 PROOFS OF SECTION 6.2

Case 2. As discussed in Section 6.2, we first construct an example where the return structure provides no information on the risk chosen by the agent. We then show that nothing can be gained by non-linear bonuses. Finally, we show that the good bank can never implement a separating equilibrium without excessive risk-taking. As the safe project's return must not be deterministic in this case, we need a slightly different setting than in our original model.

Assume that the agent is in charge of a project that can yield three returns, $R_0 = 0$, $R_1 = 100$ and $R_2 = 200$. The agent can spend an amount γ of the firm's capital to increase the probability of positive returns. For instance, γ could be expenditures for a consulting firm or for investing in a new technology. Specifically, we assume that the probabilities for positive returns are $p_i(R_1) = \frac{\theta_i}{2}\sqrt{\frac{\gamma_i}{2}}$ and $p_i(R_2) = \theta_i\sqrt{\frac{\gamma_i}{2}}$, where $i \in \{H, L\}$ and $\theta_H > \theta_L$, and that $p_i(R_0) = 1 - p_i(R_1) - p_i(R_2)$.[18] Thus, higher expenditures increase the success probabilities which are also type-dependent.[19] We furthermore assume that there is a private cost for the agent of choosing γ which is simply given by $C(\gamma) = \gamma$. This assigns a positive role to variable payments: Without bonuses, the agent will just choose $\gamma = 0$.

Note that the variance of the net return is strictly increasing in γ, so that expenditures γ measure the risk taken by the agent. Due to risk neutrality, the socially optimal risk simply maximizes the expected output, that is, the social welfare function

$$W_i(\gamma_i) = 100\frac{\theta_i}{2}\sqrt{\frac{\gamma_i}{2}} + 200\,\theta_i\sqrt{\frac{\gamma_i}{2}} - \gamma_i - C(\gamma_i) = 250\,\theta_i\sqrt{\frac{\gamma_i}{2}} - 2\gamma_i$$

which gives the first best expenditures[20]

$$\gamma_i^{\mathrm{FB}} = \frac{125^2}{8}\theta_i^2. \tag{D.8}$$

Thus, the first best expenditure (or risk) is higher for the high type.

The key features of this example are that the ratio of the probabilities for both positive returns is independent of the risk taken (γ), *and* that this ratio is the same for both agent types.[21] Specifically, we have $\dfrac{p_1(R_1)}{p_1(R_2)} = \dfrac{p_2(R_1)}{p_2(R_2)} = \dfrac{1}{2}$.

We prove that nothing can be gained by switching from linear to non-linear bonus schemes and that, just as in our main model, the excessive risk is increasing in $\widehat{U}_H^B - \widehat{U}_L^B$.[22] The good bank's expected profit in a separating equilibrium is (omitting the index G that denotes the good bank)

$$\begin{aligned}\Pi(\cdot) = \alpha &\left[100\frac{\theta_H}{2}\sqrt{\frac{\gamma_H}{2}}(1 - w_{1H}) + 200\theta_H\sqrt{\frac{\gamma_H}{2}}(1 - w_{2H}) - \gamma_H - F_H\right] \\ &+ (1-\alpha)\left[100\frac{\theta_L}{2}\sqrt{\frac{\gamma_L}{2}}(1 - w_{1L}) + 200\theta_L\sqrt{\frac{\gamma_L}{2}}(1 - w_{2L}) - \gamma_L - F_L\right].\end{aligned} \tag{D.9}$$

[18] To ensure that all probabilities are well-defined, we assume $\gamma_i \leq 8/(9\theta_H^2)$.

[19] Instead, we could assume that the investment size is fixed, and that the risk choice shifts probability from a net return of zero to both positive and negative net returns. However, results are then qualitatively the same, at the expense of more tedious calculations.

[20] We use "risk", "investment" and "expenditures" synonymously for γ as the risk is strictly increasing in the amount γ the agent spends.

[21] The example generalizes easily to a continuous version where the probability distribution over all *positive* returns is independent of risk taking.

[22] We treat $\widehat{U}_H^B - \widehat{U}_L^B$ as exogenously given and restrict attention to the good bank's best response function. Otherwise, we would have to duplicate the whole competition analysis.

Here, w_{ji} denotes the bonus fraction paid to agent i if return R_j is observed.

As in the main model, the bank needs to observe the two types' incentive compatibility- and participation constraints. Again, the bank has no incentive to deviate from the first best for the low type. An agent who accepts the contract designed for her maximizes

$$U_i = 100\frac{\theta_i}{2}\sqrt{\frac{\gamma_i}{2}}w_{1i} + 200\theta_i\sqrt{\frac{\gamma_i}{2}}w_{2i} - \gamma_i + F_i$$

and hence chooses $\gamma_i^* = \dfrac{1}{2}\theta_i^2\left(25w_{1i} + 100w_{2i}\right)^2$. An optimum requires that $\gamma^* = \gamma^{\mathrm{FB}}$, which by Equation (D.8) holds if and only if $w_{1i} + 4w_{2i} = \dfrac{5}{2}$. The linear bonus scheme leading to the first best is thus $w_{1i} = w_{2i} = \dfrac{1}{2}$.

An agent's utility is given by

$$U_i = \frac{625}{2}\theta_i^2(w_{i1} + 4w_{i2})^2 + F_i,$$

and in particular the low type's utility is $U_L = \dfrac{125^2}{8}\theta_L^2 + F_L$.

We know that *(PCH)* and *(ICCL)* are binding. By contrast, *(ICCH)* is non-binding. Thus, the relevant constraints are:

$$\begin{aligned}
\text{(PCH)} \quad &: \quad \frac{625}{2}\theta_H^2\left(w_{1H} + 4w_{2H}\right)^2 + F_H = \widehat{U}_H^B \\
\text{(ICCL)} \quad &: \quad \frac{125^2}{8}\theta_L^2 + F_L = \frac{625}{2}\theta_L^2\left(w_{1H} + 4w_{2H}\right)^2 + F_H \\
\text{(PCL)} \quad &: \quad \frac{125^2}{8}\theta_L^2 + F_L \geq \widehat{U}_L^B.
\end{aligned}$$

To prove that nothing can be gained with non-linear bonuses, it is sufficient to show that the marginal rate of substitution between w_{1H} and w_{2H} is the same in the bank's profit function and in *all* restrictions: The profit from the high type (cf. Equation (D.9)) remains constant for γ_H given if

$$w_{1H} = c - \frac{200\,\theta_H\sqrt{\frac{\gamma_H}{2}}}{100\,\frac{\theta_H}{2}\sqrt{\frac{\gamma_H}{2}}}\,w_{2H} = c - 4\,w_{2H},$$

for some constant c, which gives a marginal rate of substitution of $-\frac{dw_{1H}}{dw_{2H}} = 4$. Next, in *(PCH)*, the bank needs to ensure that $\frac{625}{2}\theta_H^2\left(w_{1H} + 4w_{2H}\right)^2 + F_H = \widehat{U}_H^B$ which gives

$$w_{1H} = \frac{\sqrt{2\,(\widehat{U}_H^B - F_H)}}{25\,\theta_H} - 4w_{2H}$$

and hence also a marginal rate of substitution of $-\dfrac{dw_{1H}}{dw_{2H}} = 4$. Therefore, in order to fulfill the high type's participation constraint, the bank needs to increase w_{1H} by four units when decreasing w_{2H} by one unit. Furthermore, the marginal rate of substitution in *(ICCL)* is also $-\dfrac{dw_{1H}}{dw_{2H}} = 4$. It follows that, starting from any linear bonus scheme, that is, where $w_{1H} = w_{2H}$, the bank cannot increase profits by switching to a different bonus scheme as only the expected bonus over all positive returns matters. When reducing w_{2H} by x units, w_{1H} must be increased by $4\,x$ units to meet the constraints, and this leaves expected profits unchanged.

The next point to note is that, as in our main model, the good bank can never implement a separating equilibrium without excessive risk-taking. From Equation (D.8) we already know that the socially optimal risk for the high type is given by $\gamma_H^{\text{FB}} = \dfrac{125^2}{8}\theta_H^2$ and from the previous calculations we know that the socially optimal linear bonus for her is given by by $w_{1H} = w_{2H} = \dfrac{1}{2}$. The high type's utility is then $\dfrac{625}{2}\theta_H^2\,(w_{1H} + 4w_{2H})^2 + F_H = \dfrac{125^2}{8}\theta_H^2 + F_H$, and since *(PCH)* is binding, we would have $F_H = \widehat{U}_H - \dfrac{125^2}{8}\theta_H^2$ when implementing the socially optimal risk for the high type. Thus, the socially optimal contracts for the high and for the low type would only differ in the fixed wage. And as the fixed wage is higher in the contract designed for the high type due to $\widehat{U}_H > \widehat{U}_L$, it follows immediately that the bad type would always imitate.

Again as in our main model, the good bank must therefore offer a bonus that induces excessive risk-taking for the high type. For any linear bonus w_H given, the high type maximizes

$$U_H = 250\,\theta_H \sqrt{\frac{\gamma_H}{2}}\, w_H - \gamma_H + F_H,$$

which yields $\gamma_H^* = \dfrac{125^2}{2} w_H^2 \theta_H^2$ and a utility of

$$U_H^* = \frac{125^2}{2} w_H^2 \theta_H^2 + F_H = \widehat{U}_H^B,$$

where the last equality follows from the binding *(PCH)*. If the low type imitates, she maximizes

$$U_L = 250\,\theta_L \sqrt{\frac{\gamma_L}{2}}\, w_H - \gamma_L + F_H,$$

which yields analogously $\gamma_L^* = \dfrac{125^2}{2} w_H^2 \theta_L^2$ and a utility of

$$U_L^{*,I} = \frac{125^2}{2} w_H^2 \theta_L^2 + F_H,$$

where superscript I denotes "imitation". Substituting for F_H yields

$$U_L^{*,I} = \widehat{U}_H^B - \frac{125^2}{2} w_H^2 \left(\theta_H^2 - \theta_L^2\right).$$

This shows that, given that *(PCH)* is binding, the imitation incentive is strictly decreasing in the bonus offered to the high type. This is equivalent to our main model.

To avoid case distinctions that would add nothing to the point we wish to make, assume that *(PCL)* is binding. Then, establishing a separating equilibrium requires that $\widehat{U}_L = \widehat{U}_L^B \geq U_L^{*,I}$, that is,

$$\widehat{U}_L^B \geq \widehat{U}_H^B - \frac{125^2}{2} w_H^2 \left(\theta_H^2 - \theta_L^2\right) .$$

Solving for w_H shows that the minimum linear bonus required to avoid imitation is

$$w_H = \left(\frac{2\left(\widehat{U}_H^B - \widehat{U}_L^B\right)}{125^2\left(\theta_H^2 - \theta_L^2\right)} \right)^{\frac{1}{2}} .$$

Hence, as in our main model, the inefficiency is increasing in the difference in the maximum utilities the two types can get in the bad bank ($\widehat{U}_H^B - \widehat{U}_L^B$), and decreasing in the productivity difference when choosing risky projects ($\theta_H^2 - \theta_L^2$).

Case 3. For the intermediate case 3, all assumptions are as in case 2 with the only difference that, while $\frac{p_H(R_1)}{p_H(R_2)} = \frac{\frac{\theta_H\sqrt{2\gamma}}{4}}{\frac{\theta_H\sqrt{2\gamma}}{2}} = \frac{1}{2}$ as before, now $\frac{p_L(R_1)}{p_L(R_2)} = \frac{\frac{\theta_L\sqrt{2\gamma}}{(4-x)}}{\frac{\theta_L\sqrt{2\gamma}}{(2+x)}} > \frac{1}{2}$. Thus, the marginal rate of substitution between bonuses for R_1 and R_2 is now different for the high and for the low type, and the difference is expressed by x. Then, the bank can clearly reduce the imitation incentive, and hence increase efficiency, by paying bonuses only for R_2. We show that non-linear bonuses can fully restore efficiency if and only if x is above some $\widetilde{x}$, that is, if the return structure is sufficiently informative.

We restrict attention again to the case where *(PCL)* is binding, so that the low type's utility without imitating is $\widehat{U}_L^B$. If the high type receives a bonus only for R_2, she maximizes

$$U_H = w_{2H}\theta_H\sqrt{\frac{\gamma_H}{2}}200 - \gamma_H + F_H$$

and hence chooses $\gamma_H^* = 2 \cdot 50^2 w_{2H}^2 \theta_H^2$, which gives her a utility of

$$U_H^* = 5000\left(\theta_H w_2^H\right)^2 + F_H.$$

When the low type imitates, she maximizes analogously

$$U_L = w_{2H}\frac{\gamma_L\sqrt{2\theta_L}}{(2+x)}200 - \gamma_L + F_H .$$

This gives $\gamma_L^* = \frac{1}{2} w_{2H}^2 \frac{200^2 \theta_H^2}{(x+2)^2}$ and a utility of

$$U_L^{*,I} = 20\,000 \frac{\left(\theta_L w_2^H\right)^2}{(2+x)^2} + F_H.$$

As *(PCH)* is binding, substituting for F_H delivers

$$U_L^{*,I} = 20\,000 \frac{\left(\theta_L w_2^H\right)^2}{(2+x)^2} - 5000 \left(\theta_H w_2^H\right)^2 + \widehat{U}_H^B.$$

Now we can analyze when a first best separating equilibrium is feasible. The optimal risk for the high type is $\gamma_H^{\mathrm{FB}} = \frac{125^2}{8} \theta_H^2$, and since the high type chooses $\gamma_H^* = 2 \cdot 50^2 w_{2H}^2 \theta_H^2$, this requires $w_{2H} = \frac{5}{8}$. Then, the high type's utility is

$$U_H^* = \frac{125^2}{8} \theta_H^2 + F_H$$

and hence $F_H = \widehat{U}_H^* - \frac{125^2}{8} \theta_H^2$. If the low type imitates, her utility is

$$U_L^{*,I} = \frac{125^2}{2} \frac{\theta_L^2}{(x+2)^2} + F_H$$

and thus

$$U_L^{*,I} = \frac{125^2}{2} \frac{\theta_L^2}{(x+2)^2} - \frac{125^2}{8} \theta_H^2 + \widehat{U}_H.$$

No imitation requires that $\widehat{U}_L^* \geq U_L^{*,I}$ or that

$$\widehat{U}_H^* - \widehat{U}_L^* \leq \frac{125^2}{8} \theta_H^2 - \frac{125^2}{2} \frac{\theta_L^2}{(x+2)^2}.$$

The derivative of the right hand side is $\frac{d\,(RHS)}{dx} = 125^2 \frac{\theta_L^2}{(x+2)^3} > 0$ which shows that the condition is relaxed if the difference in the marginal rates of substitution between the two returns for the two types captured by x is increasing. The critical $\widetilde{x}$ is given by

$$\widetilde{x} = 250 \frac{\theta_L}{\sqrt{125^2 \theta_H^2 - 8\widehat{U}_H^* + 8\widehat{U}_L^*}} - 2,$$

and for all $x < \widetilde{x}$, non-linear bonus schemes can only mitigate, but not solve the problem identified in our paper.