Optimal contracts under adverse selection, moral hazard and type-dependent reservation utilities

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Abstract

In a continuous-time setting where a risk-averse agent controls the drift of an output process driven by a Brownian motion, optimal contracts are linear in the terminal output; this result is well-known in a setting with moral hazard and – under stronger assumptions – adverse selection. Using techniques from stochastic control theory, we show that this result continues to hold when in addition reservation utilities are type-dependent. This type of problem occurs in the study of optimal compensation problems involving competing principals.

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1. Introduction

The design of compensation schemes in a principal-agent relationship typically involves a bonus component to incentivize the agent (worker) to act in the principal's (employer's) interest. The complicated, often highly non-linear form of contracts suggested by economic models is not met in reality where bonus schemes encountered are simple and often linear in the output generated by the agent. One explanation for this gap between theory and practice lies in the fact that linear compensation schemes are robust in the richer, more diffuse real environment, whereas the optimal contract in a highly stylised economic model fails at the slightest change of model assumptions or parameters.

Against this backdrop, Holmstrom and Milgrom (1987) show that the optimal contract for an agent controlling the drift of a process driven by a Brownian motion, when the principal observes only the output process (i.e., in a setting with moral hazard), is a linear function of the terminal output. Restricting the principal's observability to the terminal output, Sung (2005) extends this to a setting where the agent's capability is private information (i.e., adverse selection), so aside from incentivizing a worker, contracts are designed to screen workers according to their capabilities.

Under many circumstances, reservation levels of utilities are in addition type-dependent, e.g. Jullien (2000); Bénabou and Tirole (2015); Bannier *et al.* (2016). In other words, a worker with high abilities faces a higher outside option than a worker with low abilities. Aside from incentivizing and screening, optimal contracts offered to workers will reflect these different outside options. The purpose of this note is to show that the linearity of contracts in the Holmstrom-Milgrom model carries over to this setting. The proofs use techniques from stochastic control theory, see e.g. Fleming and Rishel (1975).

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2. The model

The setting is similar to Holmstrom and Milgrom (1987), Schaettler and Sung (1993) and Sung (2005). A risk-neutral principal employs a risk-averse agent whose preferences are expressed by CARA utility with parameter ρ, i.e., $U(x) = 1 - \mathrm{e}^{-\rho x}$. The agent's capabilities $\theta \in \{\theta_L, \theta_H\}$ are private information. The probabilities α, resp. $1-\alpha$, of meeting a H-type, resp. L-type, agent are known to the principal.

A type-k agent, by exerting effort $\mu = (\mu_t)_{t\geq 0}$, which is assumed to be bounded, controls the drift of a process with dynamics

$$\mathrm{d}Z_t = \mu_t\,\theta_k\,\mathrm{d}t + \sigma\,\mathrm{d}W_t, \quad t \geq 0,$$

with $\sigma > 0$ and where $W = (W_t)_{t\geq 0}$ is a Brownian motion independent of θ. The agent observes the Brownian motion W, so that μ is adapted to the filtration generated by the Brownian motion. The agent's effort is subject to an instantaneous cost $c(\mu_t)$, $t \geq 0$, where $c(0) = 0$, c is strictly increasing, convex and continuously differentiable. In addition, we require $c''' \geq 0$ for Proposition 2. We shall assume that $\theta_H > \theta_L$, expressing that a type-H agent generates a higher drift at equal effort cost than a type-L agent.

The principal observes the output process $Z = (Z_t)_{t\geq 0}$, but neither W nor μ, so the compensation, realised at time 1, can be contingent on $(Z_t)_{0\leq t\leq 1}$ only. At time 0, the principal offers contracts, consisting of sharing rules $\{S((Z_t)_{t\in[0,1]}, \theta_k), k \in \{H, L\}\}$, where $S(\cdot, \theta_k)$ denotes the sharing rule *designed* for a type-k agent. An agent choosing contract $S(\cdot, \theta_k)$ at time 0, receives at time 1

$$S((Z_t)_{0\leq t\leq 1}, \theta_k) - \int_0^1 c(\mu_t)\,\mathrm{d}t,$$

while the principal receives

$$Z_1 - S((Z_t)_{0\leq t\leq 1}, \theta_k).$$

3. The principal's problem

Denote by $S(\cdot, \theta_m)$ the contract designed for an m-type agent. An agent of type $k \in \{H, L\}$, when choosing contract $S(\cdot, \theta_m)$ exerts effort $\mu^{k,m} = (\mu_t^{k,m})_{t\geq 0}$ and derives certainty equivalent $w_{k,m}$ at time 1. Whenever $k = m$, we write μ^k and w_k. The principal's problem is as follows:

Problem 1. *Choose controls $\{\mu^H, \mu^L\}$ and a menu of contracts $\{S(\cdot, \theta_H),\, S(\cdot, \theta_L)\}$ maximising*

$$\alpha\,\mathbb{E}\left[Z_1^H - S((Z_t^H)_{0\leq t\leq 1}, \theta_H)\right] + (1-\alpha)\,\mathbb{E}\left[Z_1^L - S((Z_t^L)_{0\leq t\leq 1}, \theta_L)\right],$$

subject to

(1) $\mathrm{d}Z_t^k = \mu_t^k \theta_k\,\mathrm{d}t + \sigma \mathrm{d}W_t$, $k \in \{H, L\}$,

(2) $\mu_t^{k,m} \in \operatorname{argmax}_{(\mu_t)_{0\leq t\leq 1}} \mathbb{E}\left[U\left(S((Z_t)_{0\leq t\leq 1}, \theta_m) - \int_0^1 c(\mu_t)\,\mathrm{d}t\right)\right]$, *where* $\mathrm{d}Z_t = \mu_t\theta_k\,\mathrm{d}t + \sigma\,\mathrm{d}W_t$, *and* $k, m \in \{H, L\}$,

(3) $\mathbb{E}\left[U\left(S((Z_t^k)_{0\leq t\leq}, \theta_k) - \int_0^1 c(\mu_t^k)\,\mathrm{d}t\right)\right] \geq \mathbb{E}\left[U\left(S((Z_t^{k,m})_{0\leq t\leq 1}, \theta_m) - \int_0^1 c(\mu_t^{m,k})\,\mathrm{d}t\right)\right]$, *where* $\mathrm{d}Z_t^{k,m} = \mu_t^{k,m}\theta_k\,\mathrm{d}t + \sigma\,\mathrm{d}W_t$ *and* $m, k \in \{H, L\}$, (ICC)

(4) $\mathbb{E}\left[U\left(S((Z_t^k)_{0\leq t\leq}, \theta_k) - \int_0^1 c(\mu_t^k)\,\mathrm{d}t\right)\right] \geq U(w_k)$, *where* $k \in \{H, L\}$, (PC).

The first constraint defines the dynamics of the output processes of each type when choosing the contract designed for him. The second constraint expresses that agents maximise their expected utility. The third constraint, the incentive compatibility constraint (ICC), makes each agent optimally choose the contract designed from him. Finally, the fourth constraint, the participation constraint (PC), ensures that an agent contracts with the principal instead of choosing his outside option with certainty equivalent w_k (the general results do not change if it were unprofitable to attract a particular agent type).

4. The agent's choice of drift

We consider the agent's problem when faced with a menu of contracts. The following result is slightly adapted from Holmstrom and Milgrom (1987).

Theorem 1. *The adapted stochastic process $(\mu_t)_{0\le t\le 1}$ is implemented with certainty equivalent w by a type-k agent by a sharing rule $S((Z_t)_{0\le t\le 1}, \theta_k)$ only if*

$$S((Z_t)_{0\le t\le 1}, \theta_k) = w + \int_0^1 c(\mu_t)\,\mathrm{d}t + \int_0^1 \frac{c'(\mu_t)}{\theta_k}\,\mathrm{d}Z_t - \int_0^1 c'(\mu_t)\mu_t\,\mathrm{d}t + \frac{\rho}{2}\int_0^1 \left(\frac{c'(\mu_t)}{\theta_k}\right)^2 \sigma\,\mathrm{d}t. \tag{1}$$

Proof. See Theorem 6 of Holmstrom and Milgrom (1987) and Corollary 4.1 of Schaettler and Sung (1993). □

The first two terms provide a certainty equivalent of w and a direct compensation of the effort cost should the agent choose to exert effort $(\mu_t)_{0\le t\le 1}$. The third term incentivizes the agent to choose effort level μ. The last two terms compensate the agent for the mean and risk of the output process, i.e., they correspond to the certainty equivalent of the third term.

Proposition 1. *A type-k agent, when choosing the contract designed for the m-type agent, derives expected utility*

$$\mathbb{E}\left[U\left(w_m + \int_0^1 c'(\mu_t^{k,m})\mu_t^{k,m} - c(\mu_t^{k,m}) - \left(c'(\mu_t^m)\mu_t^m - c(\mu_t^m)\right)\,\mathrm{d}t\right)\right], \tag{2}$$

where $\mu^{k,m}$ denotes the k-type agent's optimal control, which solves

$$c'(\mu_t^{k,m}) = \frac{\theta_k}{\theta_m} c'(\mu_t^m). \tag{3}$$

Furthermore, the H-type agent exerts greater effort and derives greater utility from a given contract than the L-type agent.

Proof. Let $S((Z_t)_{0\le t\le 1}, \theta_m)$ be the contract designed for an m-type agent offering w_m and implementing μ^m. A type-k agent implementing μ, that is, $\mathrm{d}Z_t = \mu_t\theta_k\,\mathrm{d}t + \sigma\,\mathrm{d}W_t$, $t \ge 0$, derives expected utility

$$\mathbb{E}\left[U\left(w_m + \int_0^1 c(\mu_t^m) - c(\mu_t)\,\mathrm{d}t + \int_0^1 \frac{c'(\mu_t^m)}{\theta_m}\,\mathrm{d}Z_t - \int_0^1 c'(\mu_t^m)\mu_t^m - \frac{\rho}{2}\left(\frac{c'(\mu_t^m)}{\theta_m}\right)^2 \sigma^2\,\mathrm{d}t\right)\right],$$

which can be written as $\mathbb{E}\left[U(X_1^\mu)\right]$ with state process

$$X_u^\mu := w_m\,u + \int_0^u c(\mu_t^m) - c(\mu_t)\,\mathrm{d}t + \int_0^u \frac{c'(\mu_t^m)}{\theta_m}\sigma\,\mathrm{d}W_t + \int_0^u c'(\mu_t^m)\mu_t\frac{\theta_k}{\theta_m} - c'(\mu_t^m)\mu_t^m + \frac{\rho}{2}\left(\frac{c'(\mu_t^m)}{\theta_m}\right)^2 \sigma^2\,\mathrm{d}t.$$

By the Itô formula,

$$\mathrm{d}X_t^\mu = \left[w_m + c(\mu_t^m) - c(\mu_t) + c'(\mu_t^m)\left(\mu_t\frac{\theta_k}{\theta_m} - \mu_t^m\right) + \frac{\rho}{2}\left(\frac{c'(\mu_t^m)}{\theta_m}\right)^2\sigma^2\right]\mathrm{d}t + \frac{c'(\mu_t^m)}{\theta_m}\sigma\,\mathrm{d}W_t.$$

Define $J(t,x;\mu) = \mathbb{E}\left[U(X_1^\mu)|X_t^\mu = x\right]$, which is once (twice) continuously differentiable in t (x) (applying Dominated Convergence for differentiating inside the expectation operator), so that $J(0,x;\mu) = \mathbb{E}[U(X_1^\mu)]$ corresponds to the objective function. Because J is a martingale, the following PDE holds:

$$J_t + \frac{1}{2}\left(\frac{c'(\mu_t^m)}{\theta_m}\right)^2\sigma^2 J_{xx} + \left[w_m + c(\mu_t^m) - c(\mu_t) + c'(\mu_t^m)\left(\mu_t\frac{\theta_k}{\theta_m} - \mu_t^m\right) + \frac{\rho}{2}\left(\frac{c'(\mu_t^m)}{\theta_m}\right)^2\sigma^2\right] J_x = 0,$$

where J_t, J_x, J_{xx} denote the respective partial first- and second-order derivatives. Setting $V(t,x) = \sup_\mu J(t,x;\mu)$, the Hamilton-Jacobi-Bellman PDE is

$$V_t + \frac{1}{2}\sigma^2 V_{xx} + \sup_\mu\left[c'(\mu_t^m)\frac{\theta_k}{\theta_m}\mu_t - c(\mu_t)\right]V_x + \left[c(\mu_t^m) - c'(\mu_t^m)\mu_t^m + \frac{\rho}{2}\left(\frac{c'(\mu_t^m)}{\theta_m}\right)^2\sigma^2\right]V_x = 0,$$

with boundary condition $V(1,x) = U(x)$. The agent's Hamiltonian is given by

$$\mathcal{H} = -c(\mu_t) + c'(\mu_t^m)\mu_t\frac{\theta_k}{\theta_m},$$

leading to the optimal effort choice $\mu_t^{k,m}$ fulfilling the FOC (3).

If $k = H$ and $m = L$, the FOC expresses that the H-type agent exerts greater marginal effort on the L-type's contract than the L-type. Moreover, because of the convexity of c, it follows that $\mu_t^{H,L} > \mu_t^L$. Conversely, if $k = L$ and $m = H$, then $\mu_t^{L,H} < \mu_t^H$.

The k-type agent's expected utility is given by (2). Because c is strictly increasing and convex, the mean value theorem implies $c'(x)x > c(x)$, and

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[c'(x)x - c(x)\right] = c''(x)x > 0, \quad x > 0.$$

Therefore, when $m = H$ and $k = L$, the integral is strictly positive, and the H-type agent derives a greater utility from the L-type's contract than the L-type. Conversely, if $m = L$ and $k = H$, the L-type derives a smaller utility from the H-type's contract than the H-type. □

The classical result is obtained that if both types' reservation utilities are equal, then the H-type has an incentive to imitate if second-best contracts were offered (i.e., contracts with moral hazard only), in which case the contract designed for the L-type needs to be distorted to prevent the H-type from imitating (e.g. Salanié (2005)). If reservation utilities are type-dependent, then the situation may be reversed, and the H-type's contract may need to be distorted to prevent the L-type from imitating.

5. Optimal contracts

Turning to Problem 1, we restrict the analysis to the case where the H-type has an imitation incentive; the case when the difference of the types' reservation utilities is sufficiently large for the L-type to have an imitation incentive is treated in a similar way. We omit the proof of the following well-known results when the H-type has an imitation incentive (e.g. Salanié

(2005)): The L-type's *(PC)* is binding (constraint (4) in Problem 1; to attract the L-type) and the H-type's *(ICC)* is binding (constraint (3) in Problem 1; to prevent the high type from imitating), while the L-type's *(ICC)* is non-binding. The H-type's contract features the second-best (constant) drift rate $\mu^{H,\star}$ (there is no reason to deviate from the optimum), while the effort level $\mu^{L,\star}$ in the contract for the L-type is distorted to prevent the H-type from imitating.

The principal thus solves

$$\sup_{\mu^H,\mu^L,S(\cdot,\theta_H),S(\cdot,\theta_L)} \mathbb{E}\left[\alpha(Z_1^H - S((Z_t^H)_{0\le t\le 1},\theta^H)) + (1-\alpha)(Z_1^L - S((Z_t^L)_{0\le t\le 1},\theta^L))\right],$$

subject to

$$\text{(1)}\quad \mathbb{E}\left[U\left(S((Z_t^L)_{0\le t\le 1},\theta_L) - \textstyle\int_0^1 c(\mu_t^L)\,\mathrm{d}t\right)\right] = U(w_L); \qquad \textit{(PCL)}$$

$$\text{(2)}\quad \mathbb{E}\left[U\left(S((Z_t^{H,L})_{0\le t\le 1},\theta^L) - \int_0^t c(\mu_t^{H,L})\,\mathrm{d}t\right)\right] = \mathbb{E}\left[U\left(S((Z_t^H)_{0\le t\le 1},\theta^H) - c(\mu^{H,\star})\right)\right]$$
$$= \mathbb{E}\Big[1-\exp\Big(-\rho\underbrace{\Big(w_L + \int_0^1 c'(\mu_t^{H,L})\mu_t^{H,L} - c(\mu_t^{H,L}) - \left[c'(\mu_t^L)\mu_t^L - c(\mu_t^L)\right]\,\mathrm{d}t\Big)}_{=:u_1(\mu^L)}\Big)\Big],$$

with $\mu_t^{H,L}$ given by Equation (3); *(ICCH)*

$$\text{(3)}\quad \mathbb{E}\left[U\left(S((Z_t^H)_{0\le t\le 1},\theta^H) - c(\mu^{H,\star})\right)\right] \ge \mathbb{E}[U(w_H)]; \qquad \textit{(PCH)}$$

Proposition 2. *Under adverse selection, moral hazard and when reservation utilities are type-dependent, the optimal effort levels $\mu^{H,\star}$, $\mu^{L,\star}$ in the contracts designed for the H-type, resp. L-type agent are constant, and optimal contracts are linear in the terminal outputs Z_1^H and Z_1^L.*

In the proof it is shown that if *(PCH)* is non-binding, then $\mu^{L,\star}$ satisfies

$$c'(\mu^{L,\star}) = \left(\theta_L - \frac{\alpha}{1-\alpha}\left\{c''(\mu^{H,L,\star})\mu^{H,L,\star}\frac{\partial}{\partial\mu^{L,\star}}\mu^{H,L,\star} - c''(\mu^{L,\star})\mu^{L,\star}\right\}\right)\left(1+\frac{\rho\sigma^2}{\theta_L^2}c''(\mu^{L,\star})\right)^{-1}, \tag{4}$$

By the assumption that $c''' \ge 0$ it follows directly that the optimal control is smaller than the second best optimal control without adverse selection (which is obtained when $\alpha = 0$). If *(PCH)* is binding, then $\mu^{L,\star}$ satisfies

$$w_H - w_L = c'(\mu^{H,L,\star})\mu^{H,L,\star} - c(\mu^{H,L,\star}) - [c'(\mu^{L,\star})\mu^{L,\star} - c(\mu^{L,\star})]. \tag{5}$$

Proof. First, assume that *(PCH)* is non-binding (this case arises when the difference between reservation utilities is small or zero). From *(ICCH)*, where the right-hand side depends only on the effort level incentivized by the principal, it follows that the H-type's certainty equivalent is $-\frac{\ln\mathbb{E}[\mathbf{e}^{-\rho u_1(\mu^L)}]}{\rho}$.

Using Theorem 1, the conditions imply

$$S((Z_t^L)_{0\le t\le 1},\theta_L) = w_L + \int_0^1 c(\mu_t^L)\,\mathrm{d}t + \int_0^1 \frac{c'(\mu_t^L)}{\theta_L}\,\mathrm{d}Z_t^L - \int_0^1 c'(\mu_t^L)\,\mu_t^L\,\mathrm{d}t + \frac{\rho}{2}\int_0^1\left(\frac{c'(\mu_t^L)}{\theta_L}\right)^2\sigma^2\,\mathrm{d}t \tag{6}$$

and

$$S((Z_t^H)_{0\le 1\le t},\theta_H) \\ = \frac{-\ln \mathbb{E}[\mathbf{e}^{-\rho\, u_1(\mu^L)}]}{\rho} + c(\mu^{H,\star}) + \frac{c'(\mu^{H,\star})}{\theta_H} Z_1^H - c'(\mu^{H,\star})\mu^{H,\star} + \frac{\rho}{2}\left(\frac{c'(\mu^{H,\star})}{\theta_H}\right)^2 \sigma^2. \quad (7)$$

Setting

$$X_t^{\mu^L} := \alpha\left(Z_t^H - \left\{-\frac{\ln \mathbb{E}[\mathbf{e}^{-\rho\, u_t(\mu^L)}]}{\rho} + c(\mu^{H,\star})t + \frac{\rho}{2}\left(\frac{c'(\mu^{H,\star})}{\theta_H}\right)^2 \sigma^2\, t + \frac{c'(\mu^{H,\star})}{\theta_H}\sigma\, W_t\right\}\right) \\ + (1-\alpha)\left(Z_t^L - \left\{w_L\, t + \int_0^t c(\mu_u^L)\,\mathrm{d}u + \frac{\rho}{2}\int_0^t \left(\frac{c'(\mu_u^L)}{\theta_L}\right)^2 \sigma^2\,\mathrm{d}u + \int_0^t \frac{c'(\mu_t^L)}{\theta_L}\,\sigma\,\mathrm{d}W_t\right\}\right),$$

with $\mathrm{d}Z_t^L = \mu_t^L\,\theta_L\,\mathrm{d}t + \sigma\,\mathrm{d}W_t$ and $\mathrm{d}Z_t^H = \mu^{H,\star}\theta_H\,\mathrm{d}t + \sigma\,\mathrm{d}W_t$, the principal's problem is

$$\sup_{\mu_L} \mathbb{E}[X_1^{\mu^L}].$$

The dynamics of X^{μ^L} are

$$\mathrm{d}X_t^{\mu^L} = \alpha\Bigg(\left\{\mu^{H,\star}\theta_H + \frac{\partial}{\partial t}\frac{\ln \mathbb{E}[\mathbf{e}^{-\rho\, u_t(\mu^L)}]}{\rho} - c(\mu^{H,\star}) - \frac{\rho}{2}\left(\frac{c'(\mu^{H,\star})}{\theta_H}\right)^2\sigma^2\right\}\mathrm{d}t \\ + \left(1 - \frac{c'(\mu^{H,\star})}{\theta_H}\right)\sigma\,\mathrm{d}W_t\Bigg) \\ + (1-\alpha)\left(\left\{\mu_t^L\theta_L - w_L - c(\mu_t^L) - \frac{\rho}{2}\left(\frac{c'(\mu_t^L)}{\theta_L}\right)^2\sigma^2\right\}\mathrm{d}t + \left(1 - \frac{c'(\mu_t^L)}{\theta_L}\right)\sigma\,\mathrm{d}W_t\right),$$

and the principal's Hamiltonian is

$$\mathcal{H} = \alpha\,\frac{\partial}{\partial t}\frac{\ln \mathbb{E}[\mathbf{e}^{-\rho\, u_t(\mu^L)}]}{\rho} + (1-\alpha)\left\{\mu_t^L\theta_L - c(\mu_t^L) - \frac{\rho}{2}\left(\frac{c'(\mu_t^L)}{\theta_L}\right)^2\sigma^2\right\},$$

with

$$\frac{\partial}{\partial t}\frac{\ln \mathbb{E}[\mathbf{e}^{-\rho\, u_t(\mu^L)}]}{\rho} = -\frac{\mathbb{E}\left[\mathbf{e}^{-\rho\, u_t(\mu^L)}\left\{w_L + c'(\mu_t^{H,L})\mu_t^{H,L} - c(\mu_t^{H,L}) - [c'(\mu_t^L)\mu_t^L - c(\mu_t^L)]\right\}\right]}{\mathbb{E}[\mathbf{e}^{-\rho\, u_t(\mu^L)}]}. \quad (8)$$

Equation (8) describes the change in certainty equivalent offered to the H-type agent, including the information rent to prevent him from imitating. This is $\mathcal{F}_0$-measurable, i.e., fixed at time 0. By the principle of optimality, the optimal change in certainty equivalent does not depend on any particular time t; likewise the optimal control does not depend on the particular time t, so that Equation (8) is constant. This is fulfilled for a deterministic and constant control. A constant optimal control is necessary as well, as the Hamiltonian is optimised by a deterministic choice of μ_t^L, which is constant by the principle of optimality. Hence, (8) becomes

$$\frac{\partial}{\partial t}\frac{\ln \mathbb{E}[\mathbf{e}^{-\rho\, u_t(\mu^L)}]}{\rho} = -\left\{w_L + c'(\mu^{H,L})\mu^{H,L} - c(\mu^{H,L}) - [c'(\mu^L)\mu^L - c(\mu^L)]\right\},$$

and the Hamiltonian simplifies to

$$\mathcal{H} = -\alpha \Bigg\{ \underbrace{c'(\mu^{H,L})}_{=c'(\mu^L)\theta_H/\theta_L} \mu^{H,L} - c(\mu^{H,L}) - [c'(\mu^L)\mu^L - c(\mu^L)] \Bigg\} + (1-\alpha)\left\{ \mu^L \theta_L - c(\mu^L) - \frac{\rho}{2}\left(\frac{c'(\mu^L)}{\theta_L}\right)^2 \sigma^2 \right\}.$$

The optimum is determined via

$$\frac{\partial}{\partial \mu^L}\mathcal{H} = -\alpha \left\{ \frac{\theta_H}{\theta_L} c''(\mu^L)\mu^{H,L} \frac{\partial}{\partial \mu^L}\mu^{H,L} - c''(\mu^L)\mu^L \right\} + (1-\alpha)\left\{ \theta_L - c'(\mu^L) - \frac{\rho\,\sigma^2}{\theta_L^2} c'(\mu^L)c''(\mu^L) \right\},$$

which is zero if $\mu^{L,\star}$ fulfills Equation (4), requiring this is nonnegative. It is easily verified that this is a minimum by $c''' \geq 0$. The sharing rules (6) and (7) depend only on Z_1 instead of $(Z_t)_{0\leq t\leq 1}$ and are linear in Z_1.

If *(PCH)* is binding, then

$$\mathbb{E}[\mathbf{e}^{-\rho\, u_1(\mu^L)}] = \mathbf{e}^{-\rho\, w_H},$$

and

$$\mathrm{d}\mathbb{E}[\mathbf{e}^{-\rho\, u_t(\mu^L)}] = \mathrm{d}\mathbf{e}^{-\rho\, w_H\, t} = -\rho\, w_H\, \mathbf{e}^{-\rho\, w_H\, t}\,\mathrm{d}t.$$

The left-hand side is therefore an expectation of an exponential accruing at a constant rate. Since $u_1(\mu^L)$ is comprised of a constant and an integral with respect to time, by the principle of optimality, the exponent itself must be constant. Hence $\mu^{L,\star}$ is constant. Furthermore, $\mu^{L,\star}$ solves the binding participation constraint, which can be expressed as in Equation (5). □

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