Differentiation and Risk Aversion in Imperfectly Competitive Labor Markets

Christina E. Bannier[*], Eberhard Feess[†], Natalie Packham[‡] and Markus Walzl[§]

June 8, 2020

Abstract

This paper examines the effect of imperfect labor market competition on the efficiency of compensation schemes in a setting with moral hazard and risk-averse agents who have private information on their ability. Two heterogenous firms – characterized by vertical, respectively horizontal, differentiation – compete for agents by offering contracts with fixed and variable payments. The degree of competition then determines the structure of these contracts. Three regions can be distinguished: For low competition, low-ability agents are under-incentivized and exert too little effort. For high competition, high-ability agents are over-incentivized and bear too much risk. For a range of intermediate degrees of competition, however, agents' private information has no impact and contracts are second-best. An equilibrium where both firms are active exists only when the least-cost separating allocation (LCS) is interim efficient. If firms are only vertically differentiated, then the inferior firm is inactive in equilibrium, but its competitive threat still generates the three regions just described. Moreover, an equilibrium in which the inferior firm would not break even when attracting both agent types may exist even when the LCS is not interim efficient. We show that the degrees of vertical and horizontal differentiation have opposite impacts on the condition for interim efficiency of the LCS.

JEL Classification: D82, D86, J31, J33.

Keywords: Incentive compensation, screening, imperfect labor market competition, vertical differentiation, horizontal differentiation, risk aversion.

[*]Justus-Liebig-University Giessen, Licher Str. 62, 35394 Giessen, Germany, Phone: +49 641 99 22551, Fax: +49 641 99 22559, E-mail: Christina.Bannier@wirtschaft.uni-giessen.de

[†]Frankfurt School of Finance & Management, Sonnemannstr. 9-11, 60314 Frankfurt, Germany, Phone: +49 69 154008 398, Fax: +49 69 154008 4398, E-mail: e.feess@fs.de

[‡]Berlin School of Economics and Law, Badensche Strasse 50–51, 10825 Berlin, Germany, Phone: +49 30 30877-1369, Email: packham@hwr-berlin.de

[§]Innsbruck University and SFB63, Universitaetsstr. 15, 6020 Innsbruck, Austria, Phone: +43 (0)512 507 71011, Fax: +43 (0)512 507 71199, E-mail: markus.walzl@uibk.ac.at

1 Introduction

It is well-known that variable payments are a useful instrument for reducing moral hazard (effort perspective) and for attracting managerial talent (sorting perspective). However, with fierce labor market competition, negative side effects may outweigh the benefits from higher effort at the margin, yielding excessively high-powered contract schemes for high-ability agents. In the aftermath of the financial crisis, several reasons have been identified why the contracts for bank managers may be inefficiently high-powered. Acharya, Pagano, and Volpin (2016) show that excessively high bonuses are offered to attract potentially high-qualified managers even before their actual types are revealed, Thanassoulis (2012) point out that bonuses may allow for risk-sharing, and Bannier, Feess, and Packham (2013) demonstrate that bonuses may be used as screening devices to distinguish between managers who are more or less qualified for handling high-risk projects. In all of these papers, the variable component of the payment for high-ability workers increases with the degree of competition.

In this paper, we contribute to the literature on the link between labor market competition and compensation schemes by considering a model where firms compete for a risk-averse agent whose effort and ability are both private information. We demonstrate that there is a non-monotone relationship between the degree of competition and the welfare induced by equilibrium contracts. In particular, we show that, for an intermediate level of labor market competition, the agent's private information does not lead to an efficiency reduction compared to a setting where only the effort is private information.

For an intuition, consider first a monopsonistic employer. If the employer wants to hire even the low-ability agent (the *low* type), then it is well-known from the textbook-literature on monopolistic screening that the high-ability agent (the *high* type) can gain a positive rent from choosing the contract designed for the low type. In order to reduce the high type's rent, the monopsonistic employer offers a contract with inefficiently low piece rates for the low type as this reduces the high type's imitation incentive. The contract for the low type is thus inefficiently low-powered while the contract for the high type is second-best efficient.

Now consider the other extreme case of perfect labor market competition. Perfect competition implies that firms just break-even in equilibrium, which in turn requires that the high type gets a compensation equal to her expected output. Due to this high compensation, it is now the low type who has an imitation incentive, which leads to a mirror-imaged distortion: The contract designed for the high type now entails inefficiently high piece rates, i.e. she bears higher risk than in the second-best contract. The contract for the low type is second-best efficient.

The fact that the high type has an imitation incentive for monopsonistic screening while the low type has an imitation incentive for perfectly competitive labor markets suggests that there may be competition levels in-between for which no type has an imitation incentive. This is confirmed in our analysis, which allows us to distinguish among three regions: For low degrees of competition, there is a *quasi-monopsonistic* region where the piece rate in the low type's contract is inefficiently low in order to reduce the high type's imitation incentive. The low type's piece rate in this region increases with the degree of competition. The inefficiency is thus maximum in a monopsony. For high degrees of competition, there is a *quasi-competitive* region, in which the piece rate in the high type's contract is inefficiently high in order to reduce the low type's imitation incentive. The high type's piece rate in this region increases in the degree of competition, implying that the inefficiency is maximum for perfect competition. In-between, there is a full range of degrees of labor market competition where contracts for both types are second-best as private information regarding the agent's ability has no impact on piece rates in the optimal contract menu. This finding is due to risk aversion: For any contract given, the two types choose different effort levels and hence also bear different risk. Therefore, two second-best contracts are sufficient to avoid imitation for these intermediate degrees of labor market competition.

From an economic point of view, identifying a full range of levels of labor market competition where the agent's private information on their productivity is irrelevant can be seen as our main contribution (see the literature review below). From a more theoretical perspective, we contribute to the literature by characterizing the existence and multiplicity of pure-strategy equilibria depending on the degree and the mode of labor market competition.

In the first part of the paper, we restrict attention to *vertical differentiation* in the sense that both agent types are more productive in one of the two firms. To see the novelty of our findings for vertically differentiated labor market competition, recall from Rothschild and Stiglitz (1976) that a pure-strategy equilibrium with perfect competition exists if and only if offering both types of agents their expected output is interim efficient. Interim efficiency implies that offering the low type more than her output to reduce her imitation incentive and tolerating the ensuing necessary distortion in the high type's piece rate is not profitable. Following the literature, we refer to the contract pair where both types receive their output as the *least cost separating (LCS) allocation.* The LCS allocation is interim efficient if and only if the fraction of high types is below a critical threshold. We show that this critical threshold becomes more restrictive when the degree of competition decreases and the degree of risk aversion increases.

However, when the LCS allocation is not interim efficient, our model with only vertical differentiation may still yield pure-strategy equilibria that have not been considered in the literature yet. In these equilibria, the firm with lower productivity (the *bad* firm) offers the low type more than her output and the high type exactly her output. While such an *overbidding*-contract menu is weakly dominated as the bad firm would face losses with the low type and just breaks even with the high type, this strategy arises as the limit of undominated strategies as in Simon and Stinchcombe (1995). Observe that the bad firm has no incentive to deviate from such a contract menu as long as both agent types are employed by the good firm and if there is no other contract pair allowing to profitably attract at least one agent type.

The existence of an equilibrium with overbidding by the less productive firm requires that both agent types are effectively employed by the more productive firm (the *good* firm). While this seems to be an interesting type of equilibrium, it implies that the model with only vertically differentiated firms captures a situation with potential rather than actual competition.[1] In order to account for this, we extend our basic model to horizontally differentiated firms with the agent being randomly located on the unit interval at a distance x to the good firm and a distance $1-x$ to the bad firm with travelling costs $t \cdot x$ and $t \cdot (1-x)$. Then, both firms hire the agent whenever she is sufficiently close-by and vertical differentiation is not too large. In this case, the good and the bad firm hire both agent types with positive probability.

Our main economic insights generalize to this combination of vertical and horizontal differentiation: If the frequency of high-ability types is sufficiently low to guarantee a pure-strategy equilibrium for vertical differentiation, there also exists an equilibrium for horizontal differentiation (i.e. for transportation costs $t > 0$). Regarding equilibrium menus and welfare, more horizontal differentiation works in exactly the same way as more vertical differentiation: Differentiation increases (decreases) distortions of quasi-monopsonistic (quasi-competitive) contract menus and has a non-monotone impact on welfare. Straightforwardly, however, overbidding equilibria cannot exist when both firms hire a positive mass of agents as the less productive firm would have an incentive to withdraw its offers in order to avoid losses.

The remainder of the paper is organized as follows: Section 2 relates our work to the literature. Section 3 presents the model. Section 4 derives the good firm's best-response function. Section 5 examines existence and characterization of equilibria, most notably the effect of competition on efficiency. Here, we differentiate between the cases where the least-cost separating allocation is interim efficient in the bad firm (Subsection 5.1) and where it is not (Subsection 5.2). In Section 6, we extend the model with pure vertical differentiation between firms by horizontal differentiation. We conclude in Section 7.

2 Relation to the literature

Our paper is most closely related to Bénabou and Tirole (2016) who consider a multi-task principal-agent model with risk-neutral employees spending effort on an intrinsically motivated task and an incentivized task. Principals offer screening contracts consisting of fixed and variable components. Bénabou and Tirole (2016) demonstrate that horizontal differentiation in the labor market reduces welfare if the competition intensity is low and increases welfare if the competition intensity is high. Thus, as in our paper, a higher degree of competition increases (decreases) welfare when the high type (the low type) has the imitation incentive.

Compared to Bénabou and Tirole (2016), we derive two main new insights: First, there is just one degree of horizontal differentiation (i.e., one value of transportation costs t) that gives rise to a second-best efficient equilibrium in Bénabou and Tirole (2016). Conversely, welfare in our approach is second-best for a whole range of transportation costs. The reason is that risk aversion induces distinct second-best piece rates

[1] The model with only vertical diffeentiation hence comes close to the traditional literature on contestable markets (Baumol et al., 1982; Baumol and Bailey, 1984). This is mainly because our model satisfies the assumption of frictionless entry. A more recent literature has shown that competitive threats may indeed affect not only labor-related corporate decisions but also financial decisions such as dividend payouts, cash holdings or derivatives usage (Haushalter et al., 2007; Hoberg et al., 2014).

for agents with different levels of productivity. Second-best contracts are thus sufficient to avoid imitation for a range of offers by the competing firm. Second, our model where firms are only vertically differentiated allows for a pure-strategy equilibrium even when the LCS is not interim efficient, i.e., for larger fractions of highly productive agents.

In their seminal work on perfectly competitive insurance markets with information asymmetry and common values,[2] Rothschild and Stiglitz (1976) show that interim efficiency of the LCS is a necessary and sufficient condition for the existence of an equilibrium in pure strategies. We prove that this result extends to imperfect competition in our model setting whenever both firms are active in equilibrium. With vertical differentiation only, however, there are equilibria with an inactive firm even when the LCS is not interim efficient.

The early literature following Rothschild and Stiglitz (1976) kept the assumption of perfect competition and addressed the equilibrium non-existence problem in case the LCS is not interim efficient by allowing for mixed strategies (Dasgupta and Maskin, 1986) or by considering anticipatory or reactive strategies of principals, thereby changing the equilibrium concept (Wilson, 1977; Miyazaki, 1977; Riley, 1979). Later work has extended the time structure, allowed for dynamic interactions between competitors (Hellwig, 1987; Mimra and Wambach, 2011; Netzer and Scheuer, 2014; Handel, Hendel and Whinston, 2015) or introduced capacity constraints or nonexclusivity of contracts (Bisin and Gottardi, 1999; Inderst and Wambach, 2001; Schmidt-Mohr and Villas-Boas, 2008; Picard, 2014).[3]

While all these papers remain in a perfect competition framework, our work is more closely related to the literature that introduces exogenously given type-dependent reservation utilities (Lewis and Sappington, 1989; Maggi and Rodriguez-Clare, 1995; Jullien, 2000) which limit the principal's market power. Jullien (2000) also includes competing principals and shows that agents' information rents are non-monotonic and may vanish for interior types. The intuition for the result that contracts for intermediate types can be efficient is related to our finding and that of Bénabou and Tirole (2016) that an intermediate degree of competition restores efficiency. Our approach goes one step further compared to Jullien (2000) as, from each firm's perspective, the agents' outside options emerge endogenously from the other firm's contract offers. Thus, the outside options are not exogenous but part of the equilibrium itself.

While our paper is in the tradition of screening contracts in labor markets, screening under imperfect competition has also been studied in models of price discrimination.[4] Starting with the seminal work by Mussa and Rosen (1978) on monopolistic price discrimination, later work has introduced imperfect competition via both horizontal (Spulber, 1989; Schmidt-Mohr and Villas-Boas, 1999) and vertical differentiation (Stole, 1995) between firms. Motivated by market imperfections that are important

[2]For a discussion of agency problems with common values, where the agents' characteristics directly enter the principals' profit function, see Maskin and Tirole (1992). Pouyet, Salanié, and Salanié (2008) offer a concise explanation why the private values case leads to different results for competitive screening.

[3]For an overview of the screening and signalling literature, see Riley (2001), for a review of the more recent literature on screening in perfectly competitive markets, see also Mimra and Wambach (2014).

[4]See Stole (2007) for an overview of price discrimination in competitive settings.

from an applied point of view, the newer literature has considered many different settings: Garrett, Gomes, and Maestri (2019) examine competitive screening when agents are heterogeneously informed about the offers available on the market. Attar, Mariotti, and Salanié (2014a) and Attar, Mariotti, and Salanié (2014b) study nonexclusive contracting such that each agent can contract with several principals. Attar, Mariotti, and Salanié (2014a) demonstrate that multiple contracting in insurance markets naturally emerges in a unique, constrained-efficient equilibrium.[5] Attar, Mariotti, and Salanié (2014b) show that nonexclusivity exacerbates the adverse selection problem in the Rothschild-Stiglitz world.

3 Basic model

Firms, agents and productivity. Two risk-neutral firms $k \in \{G, B\}$ compete for a risk-averse agent. The agent's ability type $i \in \{H, L\}$ is private information, and is H (high) with probability α and L (low) with probability $1-\alpha$. The agent's effort e is unobservable, and the output of agent i when working for firm k is $\beta_k \theta_i e + \sigma Z$ where $\sigma > 0$, Z is a standard normally distributed random variable, and $\beta_k \in [0,1]$ and $\theta_i > 0$ capture the productivity relative to the firm and the agent type, respectively. We assume that $\theta_H > \theta_L$ and $\beta_G = 1 > \beta_B = \beta$. Thus, expected output depends on the agent's type via θ_i, her effort e, and the firm she works for via β_k. The agent's risk aversion is represented by an exponential utility function with constant coefficient of absolute risk aversion ρ. The agent receives a payoff P_i^k, and e^2 is the effort cost that she faces when exerting effort e, so that her utility is $U(P_i^k - e^2) = 1 - \mathbf{e}^{-\rho(P_i^k - e^2)}$. We normalize the agent's exogenous outside option $\overline{U}$ to zero.

Competition for agents. Firms compete for the agent by simultaneously offering take-it-or-leave-it screening contracts $(F_i, w_i) \in \mathbb{R} \times [0,1]$, where F_i is a fixed wage and w_i is a piece rate for type $i \in \{L, H\}$.[6]

Payoffs. Given the firms' compensation schemes, the agent's payoff P_i^k is given by

$$P_i^k := P_i^k(F, w, e) = F + w \cdot (\beta_k \theta_i e + \sigma Z) \, .$$

[5] Interestingly, equilibrium features cross-subsidization of agent types due to the use of latent contracts (Arnott and Stiglitz, 1991) that are inactive on the equilibrium path but allow to deter cream-skimming deviations.

[6] The optimality of linear sharing rules has been demonstrated by Holmstrom and Milgrom (1987) where an agent controls the drift $\mu(t)$ in the time interval $[0,1]$ of the process $\mathrm{d}Z = \mu(t)\mathrm{d}t + \sigma \mathrm{d}B$ with B a standard Brownian motion. The agent is risk averse with constant absolute risk aversion. The principal observes the path $(Z_t)_{0 \le t \le 1}$ and compensation takes place at time 1 in form of a sharing rule $s((Z_t)_{0 \le t \le 1})$ with the sharing rule agreed at time 0. In this setup, the optimal drift choice is a constant drift μ and the optimal sharing rule is a linear function of Z_1, see Theorem 7 of Holmstrom and Milgrom (1987). Sung (2005) shows that the optimal sharing rule remains linear in a setup with moral hazard and adverse selection, see Section 2 and Theorem A.2 of Sung (2005). Packham (2018) proves that the linear sharing rule is also optimal in our setting where reservation utilities are type-dependent.

As the error term Z is normally distributed, it follows from the moment-generating function of the normal distribution that the agent's expected utility is

$$\mathbb{E}\left[U(P_i^k - e^2)\right] = 1 - \mathbf{e}^{-\rho(F + w\beta_k\theta_i e - e^2 - \frac{\rho}{2}w^2\sigma^2)}.$$

Maximizing the agent's expected utility coincides with maximizing her certainty equivalent,

$$U_i^k(F, w, e) := F + w\beta_k\theta_i e - e^2 - \frac{\rho}{2}w^2\sigma^2. \tag{1}$$

The agent's effort choice is given by

$$e_i^k := e_i^k(w) = \underset{e\geq 0}{\operatorname{argmax}} \left\{F + w\beta_k\theta_i e - e^2 - \frac{\rho}{2}w^2\sigma^2\right\} = \frac{1}{2}w\beta_k\theta_i.$$

Inserting into (1) and simplifying yields

$$U_i^k(F, w) := U_i^k\left(F, w, \frac{1}{2}w\beta_k\theta_i\right) = F + \frac{w^2}{4}(\beta_k^2\theta_i^2 - 2\rho\sigma^2) \tag{2}$$

as the agent's certainty equivalent.[7] We define $\widehat{U}_i^k$ as the maximum certainty equivalent agent i can get from firm k, that is,

$$\widehat{U}_i^k := \max_{(F,w)\in\mathbf{\Omega}^k} U_i^k(F, w),$$

where $\mathbf{\Omega}^k$ denotes the set of contracts offered by principal k. We introduce the tie-breaking rule that both types accept the good firm's offer if $\widehat{U}_i^G = \widehat{U}_i^B$. In Section 4.2 we show that the results are robust with respect to other tie-breaking rules such as 50-50 tie-breaking.

Note that the marginal utility of the piece rate is higher for the high type, that is, the single-crossing property holds:

$$\frac{\partial^2 U_i^k}{\partial w \partial\theta_i} = w\beta_k^2\theta_i > 0. \tag{3}$$

Finally, firm k's expected profit from agent i is

$$\Pi_i^k(F, w) := (1 - w)\beta_k\theta_i e_i^k - F = \frac{1}{2}(1 - w)w\beta_k^2\theta_i^2 - F. \tag{4}$$

Sequence of events. We consider the following game:

- *Stage 0*: Nature chooses the agent's type which becomes private information.
- *Stage 1*: Firms simultaneously offer take-it-or-leave-it contracts to the agent.
- *Stage 2*: Depending on her type, the agent chooses her utility-maximizing contract and her effort.
- *Stage 3*: Profits and payments are realized.

[7] In the following, we use the terms certainty equivalent and expected utility interchangeably.

Complete information and the second-best piece rate. For later reference let us first consider the case without private information on types. In this case, each firm faces the usual trade-off between risk allocation and incentives and implements two second-best piece rates, which equilibrate the losses from inefficiently low effort and from insufficient risk-sharing at the margin. For each type, firm k maximizes profits as given in (4) subject to the following binding participation constraint *(PC)* defined by the maximum utility $\widehat{U}_i^{\overline{k}}$ type i could get in the competing firm $\overline{k} \neq k$:

$$F + \frac{w^2}{4}(\beta_k^2\theta_i^2 - 2\rho\sigma^2) = \widehat{U}_i^{\overline{k}}.$$

After substituting for F and simplifying, we obtain

$$\Pi_i^k(w) = \frac{1}{2}w\left(\beta_k^2\theta_i^2 - \frac{w\beta_k^2\theta_i^2}{2} - w\rho\sigma^2\right) - \widehat{U}_i^{\overline{k}},$$

and maximizing Π_i^k with respect to w yields the second-best piece rate

$$w_i^{k,sb} = \frac{\beta_k^2\theta_i^2}{\beta_k^2\theta_i^2 + 2\rho\sigma^2}\ . \tag{5}$$

Observe that $w_i^{k,sb}$ increases in the agent's type-dependent productivity θ_i and in the firm-dependent productivity β_k and decreases in the risk-aversion parameter ρ.

Welfare. In our model, different degrees of competitive pressure exerted by the bad firm will lead to different allocations of the generated surplus between firms and workers. For welfare comparisons we consider the total surplus

$$W = \sum_{k=G,B}(\Pi^k + \alpha\widehat{U}_H^k + (1-\alpha)\widehat{U}_L^k),$$

i.e., we apply the concept of Kaldor-Hicks efficiency and consider an outcome as more efficient if those who are made better off could compensate those who are made worse off, so that a Pareto improvement could be achieved. We shall see that in any equilibrium, both worker types are employed by the good firm, and at least one piece rate is second-best. Thus, our welfare criterion effectively boils down to the degree of distortion in the piece rate that is not second-best.

4 The good firm's best response

In this section, we derive basic properties of a firm's best response *if* this firm hires both types of agents. As the good firm indeed employs both types of agents in equilibrium (see Lemma 3 below) we restrict ourselves to the good firm's best response in this section and continue with the bad firm's equilibrium behavior in Section 5.

Assuming that the good firm wants to attract both agent types, its best-response function, suppressing the firm's superscript, is a solution to

$$\max_{F_H,w_H,F_L,w_L} \Pi(F_H,w_H,F_L,w_L)$$
$$= \alpha\left(\frac{1}{2}(1-w_H)\,w_H\theta_H^2 - F_H\right) + (1-\alpha)\left(\frac{1}{2}(1-w_L)\,w_L\theta_L^2 - F_L\right) \quad (6)$$

subject to the following constraints:

$$U_H(F_H,w_H) \geq \widehat{U}_H^B, \qquad \textit{(PCH)},$$
$$U_L(F_L,w_L) \geq \widehat{U}_L^B, \qquad \textit{(PCL)},$$
$$U_H(F_H,w_H) \geq U_H(F_L,w_L), \qquad \textit{(ICCH)},$$
$$U_L(F_L,w_L) \geq U_L(F_H,w_H) \qquad \textit{(ICCL)}.$$

Here, $\widehat{U}_i^B$ is the maximum utility agent type i can gain from the bad firm's contract offers. From the good firm's perspective, $\widehat{U}_i^B$ is agent i's reservation utility. As usual, $\widehat{U}_i^B$ is exogenous in the good firm's best response and endogenously determined in equilibrium.

Solutions to (6) have a concise structure.

Lemma 1. *Let* $(F_H^*,w_H^*),(F_L^*,w_L^*)$ *be a solution to (6). Then,*

1. $w_H^* \geq w_H^{sb}$ *and* $w_L^* \leq w_L^{sb}$*;*
2. *If* $w_L^* < w_L^{sb}$*, then:* (i) $w_H^* = w_H^{sb}$*;* (ii) (ICCH) *and* (PCL) *are binding, while* (iii) (ICCL) *is non-binding;*
3. *If* $w_H^* > w_H^{sb}$*, then:* (i) $w_L^* = w_L^{sb}$*;* (ii) (ICCL) *and* (PCH) *are binding, while* (iii) (ICCH) *is non-binding.*

All proofs are in the Appendix.

As a direct consequence of $w_{sb}^H > w_{sb}^L$ and Lemma 1, it follows that the solution to (6) is always a menu of contracts, not a pooling contract.

According to the Lemma, at most one piece rate will be distorted because at most one agent type has an imitation incentive when two second-best piece rates are offered. When the high type has the imitation incentive, then the low type's piece rate is distorted, her incentive compatibility constraint (ICCL) is non-binding and her participation constraint (PCL) is binding. The high type's piece rate is second-best efficient ("no distortion at the top") and her incentive compatibility constraint (ICCH) is binding (Part 2 of the Lemma). These features are known from monopsonistic screening. Part 3 states analogous features known from competitive screening where the low type has an imitation incentive resulting in a binding (ICCL) and (PCH), and a second-best piece rate for the low type ("no distortion at the bottom").

Hence, solutions to (6) are either *quasi-monopsonistic* with a distorted piece rate for low types, *quasi-competitive* with a distorted piece rate for high types, or have *second-best* piece rates for both types. The following Proposition shows that the optimality of each of these contract types depends in a monotone fashion on the *utility spread* $\Delta\widehat{U}^B := \widehat{U}^B_H - \widehat{U}^B_L$ offered by the bad firm. It is also shown in the proof of Proposition 1 that Problem (6) is concave.

Proposition 1. *Let $(F^*_H, w^*_H), (F^*_L, w^*_L)$ be a solution to (6). Then, there exist $\Delta\widehat{U}^B_{QM}$, $\Delta\widehat{U}^B_{QC} \in \mathbb{R}$ with $\Delta\widehat{U}^B_{QM} < \Delta\widehat{U}^B_{QC}$ such that:*

Region 1 (Quasi-monopsonistic, QM): *If $\Delta\widehat{U}^B < \Delta\widehat{U}^B_{QM}$, the high type's piece rate is second-best, $w^*_H = w^{sb}_H$, and* (PCL) *and* (ICCH) *are binding. The piece rate for the low type is below second-best, $w^*_L < w^{sb}_L$, and determined by:*

(a) the first-order condition of the good firm's maximization problem, in which case the high type's participation constraint, (PCH)*, is non-binding;*

(b) the binding (PCH) *otherwise.*

Both $\Delta\widehat{U}^B$ and w^*_L are greater in Region QM(b) than in Region QM(a), and w^*_L is strictly increasing in $\Delta\widehat{U}^B$ in Region QM(b). Social welfare is increasing in $\Delta\widehat{U}^B$.

Region 2 (Second-best, SB): *If $\Delta\widehat{U}^B \in \left[\Delta\widehat{U}^B_{QM}, \Delta\widehat{U}^B_{QC}\right]$, both piece rates are second-best.*

Region 3 (Quasi-competitive, QC): *If $\Delta\widehat{U}^B > \Delta\widehat{U}^B_{QC}$, the low type's piece rate is second-best, $w^*_L = w^{sb}_L$, and* (PCH) *and* (ICCL) *are binding. The piece rate for the high type is above second-best, $w^*_H > w^{sb}_H$, and determined by:*

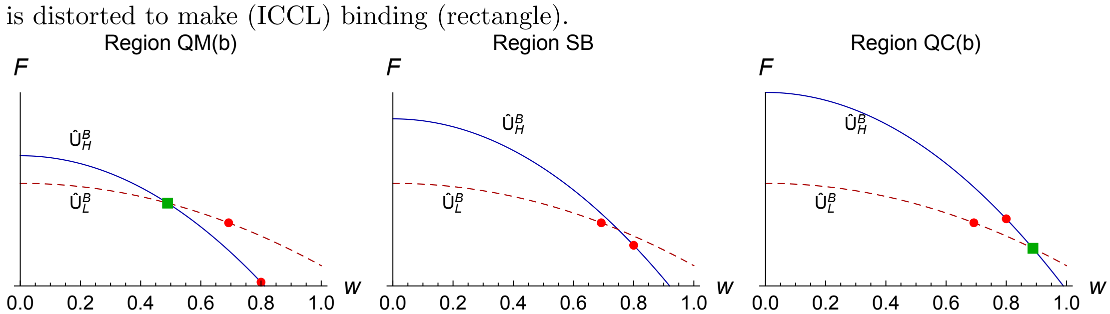
(a) the first-order condition of the good firm's maximization problem, in which case the low type's participation constraint, (PCL) *, is non-binding;*

(b) the binding (PCL) *otherwise.*

Both $\Delta\widehat{U}^B$ and w^*_H are greater in Region QC(a) than in Region QC(b) and w^*_H is strictly increasing in $\Delta\widehat{U}^B$ in Region QC(b). Social welfare is decreasing in $\Delta\widehat{U}^B$.

For a low utility spread (i.e., $\Delta\widehat{U}^B < \Delta\widehat{U}^B_{QM}$), the best response is similar to optimal contracts in a monopsony: The high type has an imitation incentive and the good firm tries to equilibrate the marginal loss from the distortion in the low type's piece rate with the marginal reduction in the high type's information rent. For $\Delta\widehat{U}^B$ sufficiently close to zero (i.e., close to the monopsony case), the high type receives an information rent that is decreasing in her reservation utility. As $\Delta\widehat{U}^B$ increases, the information rent and thereby the need to distort the low type's piece rate for rent reduction diminishes. As a result, w^*_L (and social welfare) is increasing in $\Delta\widehat{U}^B$ (see Region QM(a)). If $\Delta\widehat{U}^B$ is so

Figure 1: Best responses of good firm in region QM(b) (left), region SB (middle) and region QC(b) (right). In each graph, the solid (dashed) line denotes the high (low) type's indifference curve at the reservation level $\widehat{U}_H^B$ ($\widehat{U}_L^B$) for different combinations of fixed wages and piece rates. Points denote second-best contracts. In region QM(b), the high type has an imitation incentive when two second-best piece rates are offered as she would reach an indifference curve to the northeast when choosing the contract designed for the low type. In order to prevent imitation, the low type's piece rate is decreased to make (ICCH) binding; this is depicted by the rectangle. Likewise, in region QC(b), the low type has an imitation incentive, and the high type's contract is distorted to make (ICCL) binding (rectangle).



large that the high type does not receive an information rent (i.e., (PCH) is binding), there is no point in further reducing her imitation incentive, and $w_L < w_L^{sb}$ is determined by the binding (PCH) instead (Region QM(b)).[8]

For a high utility spread (i.e., $\Delta\widehat{U}^B > \Delta\widehat{U}_{QC}^B$), the best response is similar to optimal contracts under perfect competition: With a high utility spread offered by the bad firm, the low type has an imitation incentive whenever the good firm attracts both types. In order to reduce the low type's information rent, the contract for the high type is inefficiently high-powered. As the utility spread increases, the imitation incentive becomes larger and the corresponding distortion of the high type's piece rate (weakly) increases (and social welfare decreases, see Region QC(a)). If the efficiency loss due to distorted piece rates for the high type becomes so large that an information rent is paid to low types (i.e., (PCL) is not binding), rents for the low type (but not piece-rate distortions) are increasing in $\Delta\widehat{U}^B$ (see Region QC(b)).

For intermediate values of $\Delta\widehat{U}^B$ contracts are second best. Inefficiencies arise only from moral hazard and risk aversion, and incomplete information poses no further constraints. Neither agent type can benefit from imitating the other when both are held on their reservation utilities and both piece rates are second-best efficient. The existence of a whole *second-best region* (instead of just one utility level $\Delta\widehat{U}^B$ where this is the case) follows from the fact that the agents' second-best piece rates differ as the trade-off between effort incentives and risk aversion is type-specific. Figure 1 illustrates the three different regions.

For risk-neutral agents (i.e., $\rho = 0$), second-best piece rates are first best (i.e.,

[8] Any piece rate below would reduce the efficiency of the low type's contract, but would not allow to offer a lower overall remuneration to the high type. Consequently, the piece rate for the low type is higher in Region QM(b) than in Region QM(a).

$w_i^k = w_i^{k,fb} = 1$). With private information, the high type's piece rate is still distorted upwards for high $\Delta\widehat{U}^B$ while the low agent type's piece rate is distorted downwards for low $\Delta\widehat{U}^B$. The region where private information does not lead to a welfare reduction collapses to one point, namely where $\Delta\widehat{U}^B = \frac{1}{4}\left(\theta_H^2 - \theta_L^2\right)$. Thus, the existence of a whole region of (intermediate) degrees of competitive pressure without any impact of asymmetric information with respect to types can be attributed to agents' risk aversion.

5 Existence and characterization of equilibria

5.1 The least-cost separating allocation is interim efficient

As a starting point for the equilibrium analysis, we consider a contract menu by the bad firm where both types are offered their expected output, so that the firm breaks even when attracting both types. Following Bénabou and Tirole (2016), we refer to this contract menu as the *(bad firm's) least-cost separating allocation (LCS)*. We show that, similar to the Rothschild-Stiglitz-model of perfect competition, the bad firm's LCS allocation is part of an equilibrium if and only if the LCS allocation is interim efficient.

Definition 1. An incentive compatible contract menu $(F_i^*, w_i^*)_{i=H,L}$ offered by firm k with profit $\Pi^{k,*} \geq 0$ is **interim efficient (IE)** if there is no other incentive-compatible menu $(F_i, w_i)_{i=H,L}$ with profit Π^k that Pareto dominates it: $U_i \geq U_i^*$ for $i \in \{H, L\}$ and $\Pi^k \geq \Pi^{k,*}$ with at least one inequality being strict.

The importance of IE of the LCS allocation in our setting follows from the same observations as in the literature on competitive screening: Suppose that the bad firm offers both types their expected output and that both (PCL) and (PCH) are binding in the good firm's best response. In case the LCS is not IE, the bad firm can profitably deviate by offering more than her expected output to the low type, which allows to reduce her imitation incentive and the corresponding upwards distortion in the high type's piece rate. As this new contract menu is Pareto dominant, the increase in the high type's surplus outweighs the loss from the low type. The higher the percentage of low types (i.e., the smaller α) the larger the firm's cost from offering more to low types and the smaller the gain from reducing distortions for high types. Thus, IE of the LCS requires that α is sufficiently low.

To derive the critical proportion of high types, α_{LCS}, that just ensures IE of the LCS in the bad firm, we write the expected output of type i in firm k in response to a piece rate w as the sum of the expected payoff to the firm, Π_i and the certainty equivalent of the type, U_i^k, giving $v_i^k(w) = \frac{1}{2}\beta_k^2\theta_i^2 w(1-w) + \frac{w^2}{4}(\beta_k^2\theta_i^2 - 2\rho\sigma^2)$. Here, the first term corresponds to $\Pi_i^k + F$ and the second term is $U_i^k - F$. A menu $(F_i^*, w_i^*)_{i=H,L}$ is interim efficient if and only if the loss from relaxing the low type's incentive compatibility constraint by a marginal decrease of w_H (i.e., $(1-\alpha)\frac{w_H^k}{2}\beta_k^2(\theta_H^2 - \theta_L^2)$) exceeds the gain from an output increase due to this marginal reduction of the high type's piece rate (i.e., $-\alpha \frac{dv_H^k(w_H^k)}{dw_H^k}$)). Hence, α has to be sufficiently small to ensure

$$\alpha \frac{dv_H^k(w_H^k)}{dw_H^k} + (1-\alpha)\frac{w_H^k}{2}\beta_k^2(\theta_H^2 - \theta_L^2) \geq 0. \tag{7}$$

Denote by α_{LCS} the α that solves (7) with equality for $k = B$.

Proposition 2. (i) *The bad firm's least-cost separating allocation* $(F_i^*, w_i^*)_{i=H,L}$ *is interim efficient if and only if* $\alpha \leq \alpha_{LCS}$*;* (ii) α_{LCS} *is decreasing in* ρ*;* (iii) α_{LCS} *is increasing in* β*.*

The intuition for the impact of α on interim efficiency as depicted by *Part (i)* has been discussed above and is known from screening with perfect competition. *Part (ii)* expresses that the condition for IE of the LCS becomes more restrictive when the degree of risk aversion increases. The higher the degree of risk aversion, the higher is the low type's marginal utility from getting a larger fixed salary compared to the benefit from the larger piece rate when imitating the high type. Higher risk aversion thus makes it more profitable for the firm to offer the low type more than her expected output, so that the condition for IE of the LCS becomes more restrictive when risk aversion increases. In fact, the LCS is always interim efficient for risk-neutral agents, i.e., $\alpha_{LCS} = 1$ when $\rho = 0$.

The intuition for *Part (iii)* proceeds along the same lines: The lower the productivity level β of the bad firm, the higher is the marginal utility an agent gains from getting a higher fixed payment compared to a higher variable payment. Similar to the impact of ρ, offering the low type more than her expected output is more profitable when β is low, since even a small increase in the low type's fixed salary leads to a strong reduction in her imitation incentive. Thus, the condition of interim efficiency becomes more restrictive.

As indicated by the following proposition, IE of LCS for the bad firm is sufficient for the LCS allocation to be part of an equilibrium.

Proposition 3. *If the LCS for the bad firm is interim efficient, then the LCS offered by the bad firm and the best response by the good firm as in Proposition 1 constitute an equilibrium.*

To see why offering both types their expected output by the bad firm constitutes an equilibrium when the LCS is IE, note first that the LCS allocation offered by the bad firm uniquely determines the spread in the two types' reservation utilities from the good firm's perspective $\Delta\widehat{U}^B$. From Proposition 1, we know that three regions for the good firm's best-response function need to be distinguished. As long as the good firm's best response yields a binding (PCL), the bad firm has no profitable deviation since LCS is IE and hiring any type of agent requires to offer more than her output. This is fulfilled whenever the good firm's best response is in Regions 1,2 or QC(b) in Region 3. To see that the good firm's best response is never in QC(a) of Region 3, recall from Proposition 2 *(iii)* that IE of the LCS in the bad firm implies IE of the LCS in the good firm. So *if* the good firm offers both agent types their expected output, it does not have an incentive to offer the low type more in order to reduce piece-rate distortions for the high type. But offering the expected output is only a best response under perfect competition (i.e., $\beta = 1$) in which case (PCL) is binding and the best response is in QC(b). If the bad firm offers both types their expected output, then $\Delta\widehat{U}^B$ is increasing in β - the higher the productivity, the higher the expected output difference between the high and the low type. But as $\Delta\widehat{U}^B$ is larger in region QC(a) than in region QC(b) (see Proposition 1), the good firm's best response for $\beta < 1$ has to be in QC(b) and (PCL) is indeed binding.

For the comparative statics it suffices to observe that if the bad firm offers each type her output, $\Delta\widehat{U}^B$ is increasing in β. The larger β, the larger the utility spread offered by the bad firm and the more competitive is the best response of the good firm as specified in Proposition 1.

Proposition 4. *Suppose the LCS is IE in an open neighborhood of β. Then* $\dfrac{\partial \Delta\widehat{U}^B}{\partial\beta} \geq 0$. *Moreover, whenever the LCS is IE for β_0, β_1 with $\beta_0 < \beta_1$, then $\Delta\widehat{U}^B(\beta_0) \leq \Delta\widehat{U}^B(\beta_1)$.*

The intuition why $\Delta\widehat{U}^B$ is increasing in β is that a higher productivity also increases the difference in the two agent types' productivity, $\beta\left(\theta_H^B - \theta_L^B\right)$, and thereby also their utility difference when both are offered their expected output.[9] Regarding welfare this (together with Propositions 1 and 3) implies that if the LCS is IE at β, then there is a pure strategy equilibrium $(F_i^{k,*}, w_i^{k,*})_{i=H,L}$ such that the bad firm offers each agent type her output and *(i)* $w_H^G = w_H^{G,sb}$, $\dfrac{dw_L^G}{d\beta} > 0$, and welfare is increasing in β if the good firm's best response is in region QM(b); *(ii)* $w_H^G = w_H^{G,sb}$, $w_L^G = w_L^{G,sb}$, and welfare is independent of β if the good firm's best response is in region *SB*, *(iii)* $w_L^G = w_L^{G,sb}$, $\dfrac{dw_H^G}{d\beta} > 0$, and welfare is decreasing in β if the good firm's best response is in region QC. If β is sufficiently close to zero such that the good firm's best response is in region QM(a), the piece-rate distortion (and hence social welfare) is independent of the bad firm's behavior (and β).

5.2 The least-cost separating allocation is *not* interim efficient

In contrast to models of competitive screening, where IE of LCS is sufficient *and* necessary for the existence of a pure strategy equilibrium, our model also exhibits pure strategy equilibria if LCS is not IE (i.e., $\alpha > \alpha_{LCS}$). To streamline our analysis, we exclude that the bad firm offers *both* types more than their output, i.e., the case where it would face losses when attracting just one or both of them. If the productivity difference between the two firms is sufficiently large, the good firm would still outbid the bad firm, which would thus have no incentive to deviate. Such a contract menu by the bad

[9] The proof of the Proposition demonstrates that this direct effect is not altered by the indirect impact of β on optimal piece rates.

firm, however, is implausible as it is weakly dominated. It resembles the simplest case of a Bertrand duopoly with constant marginal costs, where the bad firm could offer any price below its own and above the competitor's marginal costs.

We do not exclude, however, the case where the bad firm offers the low type more than her expected output and the high type exactly her expected output. Such a contract menu – which we will refer to as *overbidding* – arises as the limit of undominated strategies because the bad firm could earn positive profits when offering the high type an arbitrarily smaller payoff than her expected output and when attracting this type only. Again, this is similar to Bertrand competition where charging a price equal to marginal costs is weakly dominated, but nevertheless the limit of undominated strategies (and part of the unique equilibrium in pure strategies).

We hence follow Simon and Stinchcombe (1995) and restrict ourselves to limit admissible strategies.[10]

Assumption 1. *Weakly dominated strategies are excluded, except those that are limits of undominated strategies.*

Next, recall that in region QM(a) the competitive pressure exerted by the bad firm is so small that the downwards distortion in the low type's piece rate in the good firm's best response is determined as in monopsonistic screening, i.e., the good firm offers the high type more than her reservation utility in order to reduce her imitation incentive (non-binding *(PCH)*). For this case of particularly low competitive pressure, existence of equilibrium is no concern,[11] so that we subsequently ignore this case to streamline the analysis. We then make use of the following Lemma:

Lemma 2. *In equilibrium, if* (PCH) *is binding, then (i) the bad firm offers the high type her expected output; (ii) (PCL) is binding; (iii) the bad firm offers the low type at least her expected output.*

To explain, consider first that the bad firm has a profitable deviation if it offers less than expected output to the high type (*part (i)*): If *(PCL)* is non-binding, the bad firm can simply offer slightly more to the high type. If *(PCL)* is binding (which is the case in equilibrium, see *part (ii)*), the bad firm can offer the high type more (but still less than expected output) and avoids the low type's imitation by slightly increasing the high type's piece rate. Such a profitable deviation exists for all contract menus where the bad firm would earn positive profits when attracting both types. In equilibrium, the bad firm thus needs to compete as fiercely as possible.[12] Next, if *(PCL)* were non-binding in the good firm's best response, then the bad firm could offer more to the low

[10] Simon and Stinchcombe (1995) use the same criterion and call the resulting equilibria *limit admissible*. Limit admissibility is required for infinite games; otherwise one could just exclude all weakly dominated strategies.

[11] In an equilibrium with a non-binding *(PCH)*, the bad firm's behaviour is not uniquely specified. However, the contract pair where each type receives her expected output is contained in the set of best responses, even when the LCS is not IE, as the bad firm has no deviation to profitably attract the high type within QM(a). If such a deviation exists, then it is because *(PCH)* is binding, so it is outside of Region QM(a).

[12] Observe that offering the high type more than her expected output is weakly dominated as it is always the low type who has the imitation incentive in the bad firm.

type without attracting her. This would allow to reduce the upwards distortion in the high type's piece rate, and the corresponding efficiency gain would permit to profitably attract the high type. This explains *part (ii)* of the Lemma. *Part (iii)* expresses that there may be overbidding equilibria where the bad firm offers the high type her expected output and the low type more than her expected output.

As both agent types receive at least their expected output from the bad firm, a contract menu entailing cross-subsidies amongst types cannot be part of an equilibrium: In a cross-subsidy contract, the part of the high type's output not offered yet could always be used to attract the high type and to gain positive profits at the same time, while avoiding the low type's imitation incentive by increasing the high type's piece rate, thereby leaving the low type to the good firm. At the same time, cross-subsidy menus destroy a potential equilibrium where both types get their expected output if the LCS is not IE. Thus, IE of the LCS is not only a sufficient, but also a necessary condition for an equilibrium in which the bad firm would break even when attracting both types:[13]

Corollary 1. *In a pure-strategy equilibrium with binding* (PCH)*, there is no overbidding if and only if the LCS is IE.*

One final aspect needs to be considered for the discussion of equilibrium existence in case the LCS is not IE in the bad firm, which concerns the good firm's behavior: In many monopolistic screening models, the firm offers just a single contract in order to eliminate the high type's information rent when the frequency of low types, $1-\alpha$, is below a certain threshold (e.g. Section 2.2 of Salanié (2005)). This is not the case in our model:

Lemma 3. *In any pure-strategy equilibrium, the good firm hires both agent types.*

The reasoning for Lemma 3 is as follows: As $(1-\alpha) \to 0$, the piece rate w_L in the low type's contract converges to 0, and so does hence the high type's information rent since the two types' utilities are the same for $w_L = 0$. Thus, offering two contracts or just one contract yields identical profits for $(1-\alpha) = 0$, while offering two contracts is strictly superior for $(1-\alpha) > 0$.

We are now in a position to determine conditions for the existence of an equilibrium in case the LCS is not IE in the bad firm. From Lemma 2 and Corollary 1, we know that, if the LCS is not IE, the only candidate for an equilibrium offer by the bad firm (outside of region QM(a)) is an overbidding contract menu. This contract menu needs to fulfill four requirements in order to be part of a pure-strategy equilibrium:

(i). *Binding* (PCL)*:* If (PCL) is not binding and LCS is not IE, the bad firm can attract high types only by offering more to low types and reducing the distortion for high types. Hence, the utility offered by the bad firm to the low type, U_L^B, has to exceed a threshold denoted by $\widehat{U}_{L,PCL}^B$, i.e., $U_L^B \geq \widehat{U}_{L,PCL}^B$.

[13] Again, this neglects the case where β is so low that (PCH) in the good firm's best response is non-binding.

(ii). *No profitable cross-subsidy deviation:* In an equilibrium, the bad firm must not gain from further increasing losses with low types to increase gains with high types (and profitably attracting *both* types of agents). This is achieved whenever the utility offered to low types by the bad firm in equilibrium, U_L^B, exceeds a threshold $\widehat{U}_{L,CS}^B$, i.e., $U_L^B \geq \widehat{U}_{L,CS}^B$.

(iii). *Participation by the good firm*: In equilibrium, the good firm will match the bad firm's contract offer (as required by Lemma 3) only if it earns non-negative profits with the low type. Otherwise, it offers a contract to the high type with a piece rate sufficiently large to prevent the low type from imitating, and leaves the low type to the bad firm. This requires that the utility offered by the bad firm to the low type, U_L^B, does not exceed a threshold denoted by $\widehat{U}_{L,\max}^B$, i.e., $U_L^B \leq \widehat{U}_{L,\max}^B$.

(iv). *Binding* $(ICCL)$*:* As the only benefit of increasing U_L^B is to reduce the upwards distortion in the high type's piece rate, increasing it beyond the point where the bad firm's *(ICCL)* binds even when the high type's piece rate is second-best is weakly dominated. This requires that the utility offered by the bad firm to the low type, U_L^B, does not exceed a threshold denoted by $\widehat{U}_{L,\text{no imi}}^B$, i.e., $U_L^B \leq \widehat{U}_{L,\text{no imi}}^B$.

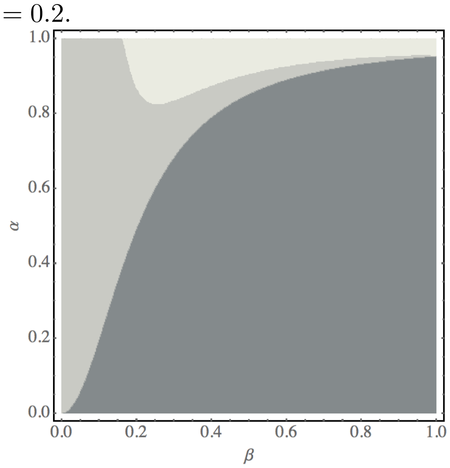
Based on these four requirements, U_L^B has to exceed a lower bound $\underline{U}_L^B$ given by $\underline{U}_L^B = \max\left(\widehat{U}_{L,PCL}^B, \widehat{U}_{L,CS}^B\right)$ and has to be below an upper bound $\overline{U}_L^B$ given by $\overline{U}_L^B = \min\left(\widehat{U}_{L,\max}^B, \widehat{U}_{L,\text{no imi}}^B\right)$ to imply equilibrium existence as summarized by the following Proposition.

Proposition 5. *(i) There is a pure-strategy equilibrium if and only if* $\underline{U}_L^B \leq \overline{U}_L^B$. *(ii) Any contract menu derived from the mutual best responses with* $U_L^B \in [\underline{U}_L^B, \overline{U}_L^B]$ *constitutes a pure-strategy equilibrium.*

Based on Proposition 5, we can now deal with the question of the impact of the competitive pressure β on the existence of overbidding equilibria. First recall that, for any α given, the condition for IE of the LCS is less restrictive if β is large, so that existence of an equilibrium without overbidding is less problematic if β is large. Such a clear-cut result, however, cannot be derived for overbidding equilibria as illustrated by Figure 2.

The dark grey area in Figure 2 shows all combinations of α and β for which the LCS in the bad firm is IE. For each α-β-pair in this area, there is an equilibrium in which the bad firm offers both agent types their expected output. Recall from Proposition 2 that IE of the LCS holds when α is sufficiently small and β sufficiently large. The figure shows that, for all other parameters given, the critical α_{LCS} increases in the bad firm's productivity β. In the medium grey area, the LCS in the bad firm is not IE, but overbidding equilibria exist. No equilibrium exists in the white area. The figure illustrates that it may well be the case that, for all other parameters kept constant, an overbidding equilibrium exists for low and for high values of β, but not for values of β in-between.

Figure 2: Existence of equilibria. In the dark grey area, the LCS is IE; equilibria exist, but the LCS is not IE in the medium grey area; no equilibria exist in the white area. Parameters are $\theta_H = 2$, $\theta_L = 0.15$, $\rho = 1$, $\sigma = 0.2$.



The intuition for the potential non-monotonicity of equilibrium existence hinges on the agents' risk aversion. Suppose first that β is very low, so that an equilibrium exists in the left part of the medium grey area in the figure. Recall that, in a potentially profitable cross-subsidy strategy, the bad firm offers the low type more than her expected output and the high type less. In doing so, however, it will never offer more to the low type than the high type's expected output. For low β, the high type's expected output in the bad firm is lower than the low type's expected output in the good firm. Therefore, the degree of overbidding required for eliminating potentially profitable cross-subsidy strategies of the bad firm does not violate the requirement that the good firm employs both types. It is therefore perfectly intuitive that overbidding equilibria exist for low levels of β, but may fail to exist when β increases.

However, there is a countervailing effect that may make the condition for equilibrium existence less restrictive as β increases. To see this, recall from our analysis of the impact of risk aversion on IE of the LCS that the second-best optimal piece rate, relative to the fixed wage, increases in β. Similar to making the condition for IE of the LCS less restrictive, a high β decreases ceteris paribus the bad firm's incentive for large cross subsidies, and a lower overbidding is hence required to eliminate the existence of a profitable cross-subsidy deviation. The non-monotonicity of equilibria existence with respect to β can thus be attributed to the fact that the impact of risk aversion is non-linear.

Finally, we turn to the characterization of overbidding equilibria with a particular focus on the impact of β on welfare. Observe from Proposition 5 that multiple equilibria exist whenever there exist values for U_L^B that are strictly above the lower and, at the same time, strictly below the upper bound. To analyze the impact of β, it is then important to be consistent in equilibrium selection. For this, we always consider the equilibrium with the lowest degree of overbidding, i.e. the equilibrium where $U_L^B = \underline{U}_L^B$.

With this criterion we select the unique perfect equilibrium in pure strategies (as defined in Simon and Stinchcombe (1995)) whenever it exists (which is the case if and only if the LCS is IE in the bad firm) and select the pure strategy equilibrium that is "closest" in terms of overbidding if no such perfect equilibrium in pure strategies exists. In Appendix A.3, we show that choosing $\overline{U}_L^B$ instead does not alter the results on the impact of β.

We extend Proposition 4 to include equilibria with overbidding.

Proposition 6. *Suppose that equilibria with* $U_L^B = \underline{U}_L^B$ *exist in a neighbourhood of* β, *and assume that* (PCH) *is binding. Then,* $\dfrac{\partial \Delta \widehat{U}^B}{\partial \beta} \geq 0$. *Moreover, if equilibria with* $U_L^B = \underline{U}_L^B$ *exist for* β_0, β_1 *with* $\beta_0 < \beta_1$, *then* $\Delta \widehat{U}^B(\beta_0) \leq \Delta \widehat{U}^B(\beta_1)$.

Proposition 6 shows that the insights for the impact of competitive pressure on the equilibrium configuration carries over from the case where the LCS is IE to the case where it is not.

6 Including horizontal differentiation

So far, we have restricted attention to vertical differentiation. This implies that the bad firm is inactive in equilibrium but shapes contracts signed by agents via its competitive pressure on the good firm. To model actual competition, we extend our basic model to horizontal differentiation (see Bénabou and Tirole, 2016) such that the good and the bad firm hire a positive mass of both types of agents in equilibrium whenever vertical differentiation is not too pronounced. Obviously, a firm hiring a positive mass of both types of agents never overbids as it would otherwise generate a loss and exiting the market would be a profitable deviation. Hence, overbidding equilibria no longer exist. As a consequence, interim efficiency of the LCS for the bad firm is not only sufficient but also necessary for the existence of a pure-strategy equilibrium. We will show that this equilibrium exhibits the same comparative statics of contracts and welfare with respect to differentiation (now vertical *and* horizontal) as discussed in Proposition 4. However, we will also prove that the higher the degree of horizontal differentiation, the less restrictive is the condition that the LCS is IE (while the opposite holds for vertical differentiation as indicated by Proposition 2).

Best responses. The extended model only differs in the assumption that agents are uniformly distributed on the unit interval and the two firms sit on each of its end-points.[14] Without loss of generality suppose that the good firm (G) is located in $x = 0$ and the bad firm (B) is located in $x = 1$. An agent located at $x \in [0, 1]$ must travel distance x (with cost tx) to firm G and distance $(1 - x)$ (with cost $t(1 - x)$) to firm B. Not considering the vertical differentiation via β, horizontal differentiation (i.e., $t > 0$) resembles a monopsony at $t \to \infty$ and perfect competition at $t \to 0$. We follow

[14] Following Bénabou and Tirole (2016) we henceforth discuss a continuum of agents. This is equivalent to considering a single agent whose location is uniformly distributed over the unit interval.

Bénabou and Tirole (2016) by assuming that the agent's exogenous outside option $\overline{U}$ (which is normalized to zero as in previous sections) also waits on the endpoints of the interval, i.e., agents must "go and get it" paying tx or $t(1-x)$. As travelling costs are additive, the agent's effort choice and expected utility (net of travelling costs) from a given contract remains unchanged and she decides to be hired by the firm that offers the larger expected utility with random tie-breaking.

Denote by $x_i^k \in [0,1]$ the distance between firm $k \in \{G, B\}$ and the agent of type $i \in \{H, L\}$ who is indifferent between the contract offer of the two firms, adopting the convention that $x_i^k = 1$ if all agents of type i prefer the contract offered by firm k and $x_i^k = 0$ if all agents of type i prefer the contract offer by the other firm. We divide firm k's profit Π_i^k from agent type i when offering a contract (F_i^k, w_i^k) into the utility she offers to the agent, U_i^k, and the surplus generated by the agent's effort in response to a piece rate w_i^k, denoted by $v_i^k(w_i^k)$, i.e.

$$\Pi_i^k = \frac{1}{2}(1-w_i^k)w_i^k\beta_k^2\theta_i^2 - F_i^k \equiv v_i^k(w_i^k) - U_i^k$$

with $v_i^k(w_i^k) = \frac{1}{2}\beta_k^2\theta_i^2 w_i^k(1-\frac{w_i^k}{2}) - \frac{(w_i^k)^2}{2}\rho\sigma^2$. β is defined as in our basic model and captures the productivity difference between the two firms. Note that $v_i^k(w_i^k)$ is monotone increasing (decreasing) for $w_i^k < (>) w_i^{k,sb} = \frac{\beta_k^2\theta_i^2}{\beta_k^2\theta_i^2+2\rho\sigma^2}$, $\frac{dv_i^k(w_i^k)}{dw_i^k} = 0$ for $w_i^k = w_i^{k,sb}$ and $\frac{d^2v_i^k(w_i^k)}{d(w_i^k)^2} < 0$. With $\widehat{U}_i^{\bar{k}}$ as the utility offered by the competing firm to type i (and multiplying the objective function by $2t$), firm k's optimization program reads

$$\max_{U_H^k, w_H^k, U_L^k, w_L^k} 2t\alpha\, x_H^k\,(v_H^k(w_H^k) - U_H^k) + 2t(1-\alpha)\, x_L^k\,(v_L^k(w_L^k) - U_L^k), \tag{8}$$

subject to

$$U_L^k \geq U_H^k - \frac{(w_H^k)^2}{4}\beta_k^2(\theta_H^2 - \theta_L^2) \qquad \textit{(ICCL)},$$

$$U_H^k \geq U_L^k + \frac{(w_L^k)^2}{4}\beta_k^2(\theta_H^2 - \theta_L^2) \qquad \textit{(ICCH)}.$$

For $U_i^k - \widehat{U}_i^{\bar{k}} < -t$ firm $\bar{k}$ is more attractive for type i than firm k regardless of the location and $x_i^k = 0$, for $U_i^k - \widehat{U}_i^{\bar{k}} > t$ firm k is more attractive for type i than firm $\bar{k}$ regardless of the location and $x_i^k = 1$. If $U_i^k - \widehat{U}_i^{\bar{k}} \in [-t, t]$, $x_i^k = \frac{1}{2} + \frac{U_i^k - \widehat{U}_i^{\bar{k}}}{2t}$. We refer to a best response that satisfies $0 < x_i^k < 1$ as an *interior best response.* As the definition of the critical location x_i^k for an interior best response implies $2tx_i^k = (t + U_i^k - \widehat{U}_i^k)$, the objective function in this case simplifies to

$$\Pi^k = \alpha(t + U_H^k - \widehat{U}_H^k)\Pi_H^k + (1-\alpha)(t + U_L^k - \widehat{U}_L^k)\Pi_L^k.$$

We first discuss optimal contracts *if* a firm wishes to hire a positive mass of both types of agents and then discuss conditions for this to be part of an equilibrium.

Structure of optimal contracts. If it is optimal for firm k to hire a positive mass of both types of agents, its best response to contract offers by the competing firm has the structure indicated by Lemma 1 for the purely vertically differentiated model (for full detail see Lemma A.1 in Appendix A.4). As a result, the same three different types of contract menus as in the purely vertically differentiated case can be optimal interior best responses:[15]

- *Quasi-monopsonic (QM):*
  $w_L^{k,*} < w_L^{k,sb}$, $w_H^{k,*} = w_H^{k,sb}$, $U_H^k - U_L^k = \frac{(w_L^{k,*})^2}{4}\beta_k^2(\theta_H^2 - \theta_L^2)$;
- *Second-best (SB):*
  $w_L^{k,*} = w_L^{k,sb}$, $w_H^{k,*} = w_H^{k,sb}$, $\frac{(w_L^{k,sb})^2}{4}\beta_k^2(\theta_H^2 - \theta_L^2) \leq U_H^k - U_L^k \leq \frac{(w_H^{k,sb})^2}{4}\beta_k^2(\theta_H^2 - \theta_L^2)$;
- *Quasi-competitive (QC):*
  $w_L^{k,*} = w_L^{k,sb}$, $w_H^{k,*} > w_H^{k,sb}$, $U_H^k - U_L^k = \frac{(w_H^{k,*})^2}{4}\beta_k^2(\theta_H^2 - \theta_L^2)$;

In particular, optimal piece rates $w_i^{k,*}$ are either second-best or $(w_i^{k,*})^2$ is proportional to $\Delta U^k = U_H^k - U_L^k$.

Monotone best responses. Also the monotone comparative statics of best-response contracts as indicated by Proposition 1 extend to the horizontally differentiated model. E.g., the utility difference offered by firm k, $\Delta U^k = U_H^k - U_L^k$, is monotone increasing in the utility difference offered by k's competitor $\Delta \widehat{U}^{\bar{k}} = \widehat{U}_H^{\bar{k}} - \widehat{U}_L^{\bar{k}}$ (for a formal statement see Proposition A.1 in Appendix A.4).

No exclusion. In contrast to the model with competitive pressure (see Lemma 3), also the bad firm hires a positive mass of both types of agents in a pure-strategy equilibrium as long as horizontal differentiation is present (i.e., $t > 0$) and vertical differentiation is not too pronounced (i.e., β is in a sufficiently small open neighborhood of 1).

Lemma 4. *(i) In any pure-strategy equilibrium, the good firm hires a positive mass of both types, i.e.* $x_i > 0$ *for* $i = H, L$.
(ii) For $t > 0$ *there is* $\beta_t < 1$ *such that the bad firm hires a positive mass of both types of agents if and only if* $\beta > \beta_t$.

Interim efficiency of the LCS and equilibrium existence. For the purely vertically differentiated model, Section 5.1 established interim efficiency of the LCS as a sufficient condition for the existence of a pure-strategy equilibrium and a necessary condition for a pure-strategy equilibrium without overbidding (unless β is so small that the best response of the good firm lies in region QM(a)). For this finding it was a crucial

[15] By the definition of an interior best response, the participation constraint for the agent of type i located at x_i^k is binding such that no distinction between quasi-competitive and quasi-monopsonic regions with and without a binding participation constraint needs to be made. For a formal derivation see Appendix A.4.

observation that with IE of the LCS, the bad firm had no incentive to offer the low type more than her output in order to reduce the inefficiency for high types. For perfect competition (i.e., $\beta = 1$ and $t = 0$) this implies that no firm has an incentive to offer more than the reservation utility (which is the expected output in this case) to the low type, i.e., the low type does not receive a rent. For $t > 0$, paying no rent to low types means that the low type agent who has to travel a distance of 1 to the firm does not receive more than her reservation utility, i.e., $U_L^k > \widehat{U}_L^k + t$. Bénabou and Tirole (2016) refer to this condition as "no cornering" of low types. As indicated by the following Lemma, IE of the LCS indeed implies the absence of cornering incentives.

Lemma 5. *For $t > 0$ there is $\beta_t < 1$ such that for all $\beta_t < \beta < 1$: If the LCS allocation $(F_i^k, w_i^k)_{i=H,L}$ is interim efficient in the bad firm for $t = 0$, i.e.,*

$$\alpha \frac{dv_H^B(w_H^B)}{dw_H^B} + (1-\alpha)\frac{w_H^B}{2}\beta^2(\theta_H^2 - \theta_L^2) \geq 0$$

then $U_i^k \leq \widehat{U}_i^{\bar{k}} + t$ for all $i \in \{H, L\}$ and $k \in \{G, B\}$.

If cornering is not optimal (as can be ensured by IE of the LCS in the bad firm for $t = 0$), best responses U_L^k and ΔU^k are continuous and strictly monotone increasing functions of $\widehat{U}_L^{\bar{k}}$ and $\Delta\widehat{U}^{\bar{k}}$ (see Proposition A.1). As the strategy space is a lattice, this implies by Milgrom and Roberts (1994, Theorem 3) the existence of a pure strategy equilibrium $C^* = (F_i^{k,*}, w_i^{k,*})$ that resembles a simultaneous solution to the optimization problem (8) for both firms. Now it is easy to see that IE of the LCS for $t = 0$ is sufficient but not necessary for the existence of a pure-strategy equilibrium: Cornering is profitable for firm k (in response to an offer according to C^* by firm $\bar{k}$ if and only if the marginal gain from reducing the piece-rate distortion in $w_H^{k,*}$ (i.e., $-\alpha x_H^{k,*} {v_H^k}'(w_H^{k,*})$) does not exceed the marginal costs of relaxing the incentive compatibility constraint for low types (i.e., $(1-\alpha)\frac{w_H^{k,*}}{2}\beta_k^2(\theta_H^2 - \theta_L^2)$ (for a formal derivation see the proof of Lemma 5), i.e.,

$$\alpha x_H^{k,*} {v_H^k}'(w_H^{k,*}) + (1-\alpha)\frac{w_H^{k,*}}{2}\beta_k^2(\theta_H^2 - \theta_L^2) \geq 0. \tag{9}$$

Denote the unique α that solves (9) with equality by α_{LCS}^*. As marginal gains from reducing the piece-rate distortion are proportional to the mass of high type agents $x_H^{k,*}$ hired by firm k and $w_H^{k,*}$ is decreasing in t (see Proposition A.2), $\alpha_{LCS}^* > \alpha_{LCS}$ for $\beta < 1$ sufficiently large to ensure interior best responses. As stated by the following Proposition, no equilibrium in pure strategies can contain cornering such that the no-cornering condition (i.e., $\alpha < \alpha_{LCS}^*$) is not only sufficient but also necessary for the existence of a pure-strategy equilibrium. We summarize as follows:

Proposition 7. *For $t > 0$ there is $\beta_t < 1$ such that for all $\beta_t < \beta < 1$, there is a pure strategy equilibrium if and only if $\alpha < \alpha_{LCS}^*$.*

The necessity and sufficiency of condition (9) extends the necessity and sufficiency of IE of the LCS for the existence of a pure-strategy equilibrium in a model with $\beta = 1$

and $t = 0$ to a model with horizontal and vertical differentiation provided that vertical differentiation is sufficiently weak such that both firms hire both types of agents in equilibrium. As the gains from cornering L types are decreasing in β and increasing in w_H, bad firms have a larger incentive to corner L types and are therefore – as in the purely vertically differentiated model – pivotal for the existence of a pure-strategy equilibrium without overbidding. Thus, while pure vertical differentiation allows for equilibria with overbidding, vertical differentiation reduces the support of pure-strategy equilibria where both firms are active. In contrast, horizontal differentiation reduces w_H^* (for a proof see Proposition A.2 in Appendix A.4) and thereby cornering incentives. The larger the differentiation as expressed through transportation costs t, the larger the support of pure-strategy equilibria.

Comparative statics. As best responses are not only continuous and monotone in the contract offer of the competitor but also in the degree of vertical and horizontal differentiation (β and t), Milgrom and Roberts (1994, Theorem 3) implies that the pure-strategy equilibrium in Proposition 7 also exhibits monotone comparative statics with respect to differentiation (for a formal proof see Proposition A.2 in Appendix A.4).

Hence, equilibria in the horizontally differentiated model with both firms hiring both types of agents share all crucial comparative statics with the purely vertically differentiated model. Provided that, in this extended model, vertical differentiation does not prevent the bad firm from hiring at all, the continuity and strict monotonicity of best responses also implies that equilibria vary in a continuous and monotone fashion in t. Critical threshold differentiations $0 < t_1 < t_2$ exist such that equilibrium contracts are quasi-monopsonistic for $t > t_2$, second-best for $t_1 \leq t \leq t_2$, and quasi-competitive for $t < t_1$ (for full detail see the proof of Proposition A.2 in Appendix A.4).

7 Conclusion

Our analysis of optimal compensation schemes for imperfect labor market competition reveals the following three insights: First, vertical and horizontal differentiation have the same impact on welfare distortions of optimal compensation schemes. The more intense competition, the more efficient (i.e., less underpowered) are incentives for low-ability agents and the less efficient (i.e., more overpowered) are incentives for high-ability agents. These results support current findings that fierce labor market competition induces inefficiently high-powered incentive contracts: Due to severe competition for managerial talent, firms have incentives to offer variable payments to high-ability agents that are above the second-best levels. Remarkably, this result is obtained without introducing limited liability or externalities, that is, even without the factors usually blamed for excessive performance pay, for instance in the financial industry. Specifically, these findings demonstrate the robustness of the non-monotone relation between competition intensity and welfare discussed by Bénabou and Tirole (2016) for different types of differentiation, risk-averse agents, and in the absence of non-contractible tasks.

Second, lessening competition through horizontal or vertical differentiation has very different implications regarding the distribution of high- and low-ability agents for which pure strategy equilibria exist. While horizontal differentiation lowers the threshold fraction of low-ability types that is necessary and sufficient for an equilibrium and moderates the problem of equilibrium existence known from competitive screening models, vertical differentiation tightens this threshold for equilibria without overbidding. However, vertical differentiation allows for the existence of overbidding equilibria (i.e., equilibria where bad firms are inactive but exercise competitive pressure) that do not exist under perfect competition and for fractions of low-ability types that prevent equilibria for competitive screening. Hence, differentiation "stabilizes" the strategic interaction in screening markets - but different types of differentiation do so via very different mechanisms.

Third, the comparative statics of optimal compensation schemes with respect to different types of differentiation and equilibrium existence interacts with the agent's risk aversion in a subtle way that has not been recognized by the literature on screening. On the one hand, risk aversion generates and enlarges a plateau of degrees of competition for which incomplete information regarding the ability of the agents does not generate distortions of optimal compensation schemes. On the other hand, risk aversion makes it more difficult to sustain an equilibrium without overbidding and/or hiring by good and bad firms since the interim efficiency of the least-cost separating allocation requires a larger fraction of low-ability types. Moreover, risk aversion induces a non-monotone impact of (vertical) differentiation on the existence of equilibria with overbidding. Hence, risk aversion reduces the frictions of incomplete information *if* an equilibrium exists but may shrink the parameter regions that support its existence.

A Formal statements and proofs

A.1 The good firm's best response

Proof of Lemma 1.

1. Deviating from second-best piece rates prevents the respective other agent type from imitating. Both properties, $w_H^* \geq w_H^{sb}$ and $w_L^* \leq w_L^{sb}$, essentially follow from the single-crossing property; that is, increasing the piece rate for the high type, resp. decreasing the piece rate for the low type, increases the utility spread between the low and the high type, which in turn prevents imitation.

Turning to the formal derivation of $w_H^* \geq w_H^{sb}$, consider a set of contracts satisfying all constraints with $w_H^* < w_H^{sb}$. We show that there exists a contract pair satisfying all constraints with piece rate w_H^{sb} that yields higher profit. From the conditions for *(PCH)* and *(ICCH)* we take $U_H(F_H^*, w_H^*) = \max(\widehat{U}_H^B, U_H(F_L^*, w_L^*))$, as any greater utility offered to the high type cannot be optimal. Offering w_H^{sb} and $F_H^{sb} := U_H(F_H^*, w_H^*) - U_H(0, w_H^{sb})$ ensures that *(PCH)* and *(ICCH)* still hold. *(PCL)* does not depend on (F_H^*, w_H^*), so it remains fulfilled. To see that *(ICCL)* still holds, observe that $U_L(F_H^{sb}, w_H^{sb}) = U_H(F_H^*, w_H^*) - \frac{{w_H^{sb}}^2}{4}(\theta_H^2 - \theta_L^2) < U_H(F_H^*, w_H^*) - \frac{{w_H^*}^2}{4}(\theta_H^2 - \theta_L^2) = U_L(F_H^*, w_H^*)$. The principal's profit is increasing in w_H for $w_H < w_H^{sb}$, (see the case with complete information case in Section 3). Since the contract for the low type is unchanged, expected profits are higher than for $w_H^* < w_H^{sb}$.

For $w_L^* \leq w_L^{sb}$, the proof is similar.

2. This is the case where the high type has an imitation incentive if second-best contracts are offered. The standard result holds that the low type's piece rate is distorted to prevent the high type from imitating. The proof is is similar to the proof of part 3 which follows.

3. This it the case where the low type has an incentive to imitate if two second-best contracts were offered. The distortion in the high type's piece rate makes *(ICCL)* binding and prevents the low type from imitating.

Formally, for *(i)*, we show that a set of contracts including $w_L^* = w_L^{sb}$ is optimal. Setting $w_L^* = w_L^{sb}$, choose $F_L^* := \max(\widehat{U}_L^B, U_L(F_H^*, w_H^*)) - \frac{(w_L^*)^2}{4}\left(\theta_L^2 - 2\rho\sigma^2\right)$, which implies $U_L(F_L^*, w_L^*) = \max(\widehat{U}_L^B, U_L(F_H^*, w_H^*))$. Hence, *(PCL)* and *(ICCL)* are fulfilled. Suppose that (F_H^*, w_H^*) are such that *(PCH)* is fulfilled. It remains to show that *(ICCH)* is fulfilled, that is $U_H(F_H^*, w_H^*) \geq U_H(F_L^*, w_L^*)$. Suppose first that *(ICCL)* is binding, $U_L(F_L^*, w_L^*) = U_L(F_H^*, w_H^*)$. Then, using $F_L^* = U_L(F_H^*, w_H^*) - U_L(0, w_L^*)$,

$$\begin{aligned}
U_H(F_H^*, w_H^*) - U_H(F_L^*, w_L^*)& \\
&= F_H^* + U_H(0, w_H^*) - F_H^* - U_L(0, w_H^*) + U_L(0, w_L^*) - U_H(0, w_L^*) \\
&= U_H(0, w_H^*) - U_L(0, w_H^*) - (U_H(0, w_L^*) - U_L(0, w_L^*)) \\
&= \frac{(w_H^{*2} - w_L^{*2})(\theta_H^2 - \theta_L^2)}{4},
\end{aligned}$$

and *(ICCH)* holds since $w_H^* \geq w_H^{sb} \geq w_L^{sb} \geq w_L^*$, as the second-best piece rate increases in θ_i. Now suppose that *(PCL)* is binding instead of *(ICCL)*, that is, $U_L(F_L^*, w_L^*) = \widehat{U}_L^B$, so that $F_L^* = \widehat{U}_L^B - U_L(0, w_L^*)$. Then, together with $U_H(F_H^*, w_H^*) \geq \widehat{U}_H^B$, we have

$$\begin{aligned} U_H(F_H^*, w_H^*) - U_H(F_L^*, w_L^*) &\geq \widehat{U}_H^B - \widehat{U}_L^B + U_L(0, w_L^*) - U_H(0, w_L^*) \\ &= \widehat{U}_H^B - \widehat{U}_L^B - \frac{w_L^{*2}}{4}(\theta_H^2 - \theta_L^2), \end{aligned}$$

which is strictly positive if and only if $\Delta\widehat{U}_B \geq \frac{w_L^{*2}}{4}(\theta_H^2 - \theta_L^2)$. By definition of the case considered, we have $w_H^* > w_H^{sb}$, since *(ICCL)* is violated if the high type is offered w_H^{sb} and F_H^{sb} such that $U_H(F_H^{sb}, w_H^{sb}) = \max(\widehat{U}_H^B, U_H(F_L^*, w_L^*))$. The violated *(ICCL)*-condition can be re-written as $\Delta\widehat{U}_B > \frac{w_H^{sb^2}}{4}(\theta_H^2 - \theta_L^2)$, and the claim then follows since $w_H^{sb} \geq w_L^*$. Finally, recall from the complete information case that this choice of (F_L^*, w_L^*) maximizes the principal's profit from the low type.

For *(ii)*, we show first that *(PCH)* is binding. Assume a contract $(F_H, w_H), (F_L, w_L)$ where *(PCH)* is non-binding and for which *(ICCH)* and *(PCH)* are fulfilled. We show that this contract is not optimal. Choose $\overline{w}_H = w_H$ and $\overline{w}_L = w_L$ and set

$$\overline{F}_H = \widehat{U}_H^B - \frac{\overline{w}_H^2}{4}(\theta_H^2 - 2\rho\sigma^2), \tag{10}$$

$$\overline{F}_L = \max(U_L(\overline{F}_H, \overline{w}_H), \widehat{U}_L^B) - \frac{\overline{w}_L^2}{4}(\theta_L^2 - 2\rho\sigma^2). \tag{11}$$

By construction, $(\overline{F}_H, \overline{w}_H), (\overline{F}_L, \overline{w}_L)$ fulfills *(PCH), (PCL)* and *(ICCL)*. For *(ICCH)* observe first that if *(PCL)* is binding, then

$$U_H(\overline{F}_H, \overline{w}_H) - U_H(\overline{F}_L, \overline{w}_L) = \widehat{U}_H^B - \widehat{U}_L^B - \frac{\overline{w}_L^2}{4}(\theta_H^2 - \theta_L^2),$$

which is strictly positive if and only if $\Delta\widehat{U}_B \geq \frac{w_L^2}{4}(\theta_H^2 - \theta_L^2)$. This holds as we consider the case where $w_H^* > w_H^{sb}$; see the proof of (i). Now suppose that *(ICCL)* is binding. Then,

$$U_H(\overline{F}_H, \overline{w}_H) - U_H(\overline{F}_L, \overline{w}_L) = \frac{w_H^2 - w_L^2}{4}(\theta_H^2 - \theta_L^2) \geq 0.$$

The so-constructed contracts yield higher profits, since the only change is that both fixed wages $\overline{F}_H$ and $\overline{F}_L$ are smaller than F_H and F_L, respectively, cf. Equations (10) and (11).

To show that *(ICCL)* is binding, we use that *(PCH)* is binding, and re-write *(ICCL)* as

$$U_H(F_H, w_H) - U_L(F_H, w_H) \geq \widehat{U}_H^B - U_L(F_L, w_L). \tag{12}$$

For the principal it is optimal to choose $w_H \geq w_H^{sb}$ as small as possible. Furthermore, by the single-crossing property, Equation (3), the left-hand side of Equation (12) is increasing in w_H. Hence, it is optimal to choose w_H such that *(ICCL)* is binding.

For *(iii)*, that is *(ICCH)* non-binding, observe that if *(ICCH)* is binding, then together with the binding *(ICCL)* we have

$$U_H(F_H, w_H) - U_H(F_L, w_L) = F_H - F_L + \frac{w_H^2 - w_L^2}{4}(\theta_H^2 - 2\rho\sigma^2) = 0$$
$$U_L(F_H, w_H) - U_L(F_L, w_L) = F_H - F_L + \frac{w_H^2 - w_L^2}{4}(\theta_L^2 - 2\rho\sigma^2) = 0.$$

This implies $\frac{w_H^2 - w_L^2}{4}(\theta_H^2 - \theta_L^2) = 0$ which requires $w_H = w_L$, i.e., a pooling contract. But this cannot be, as w_L is the second-best piece rate and $w_H \geq w_H^{sb} > w_L$, since the second-best optimal piece rate is increasing in θ_i (cf. Section 3). □

Proof of Proposition 1*.* If the good firm offers two second-best piece rates and holds both agents on their exit options, the low type has an imitation incentive, whenever

$$U_L(F_H, w_H) > U_L(F_L, w_L) = \widehat{U}_L^B,$$

which can be rewritten as

$$\widehat{U}_H^B - \widehat{U}_L^B > \frac{\theta_H^4(\theta_H^2 - \theta_L^2)}{4(\theta_H^2 + 2\rho\sigma^2)^2} =: \Delta\widehat{U}_{QC}^B,$$

i.e., when the difference in the utilities the bad firm offers to the high and low types is sufficiently large. In turn, the high type has an imitation incentive, whenever

$$U_H(F_L, w_L) > U_H(F_H, w_H) = \widehat{U}_H^B,$$

which can be rewritten as

$$\widehat{U}_H^B - \widehat{U}_L^B < \frac{\theta_L^4(\theta_H^2 - \theta_L^2)}{4(\theta_L^2 + 2\rho\sigma^2)^2} := \Delta\widehat{U}_{QM}^B,$$

i.e., when the utility difference, $\widehat{U}_H^B - \widehat{U}_L^B$, offered by the bad firm is sufficiently small. It is easily verified that $\Delta\widehat{U}_{QC}^B \geq \Delta\widehat{U}_{QM}^B$ (with equality only if $\rho\sigma^2 = 0$), so that no agent has an imitation incentive if $\Delta\widehat{U}^B \in [\Delta\widehat{U}_{QM}^B, \Delta\widehat{U}_{QC}^B]$. In this case it is of course optimal for the good firm to offer second-best piece rates to each agent. This fixes region SB.

Next, we consider the quasi-competitive region QC, in which the low type has an imitation incentive; this corresponds to Part 3 of Lemma 1. The proof for the quasi-monopsonistic region QM is similar.

We can express the good firm's maximization problem as a function of just one variable, w_H. Since the piece rate for the low type is second best, and together with the binding *(PCH)* and *(ICCL)* conditions, we obtain

$$\begin{aligned} F_H(w_H) &= \widehat{U}_H^B - U_H(0, w_H) \\ F_L(w_H) &= U_L(F_H(w_H), w_H) - U_L(0, w_L^{sb}) \\ &= \widehat{U}_H^B - U_H(0, w_H) + U_L(0, w_H) - U_L(0, w_L^{sb}). \end{aligned}$$

The good firm hence solves

$$\max_{w_H} \Pi(w_H) = \alpha\left(\frac{1}{2}(1-w_H)w_H\theta_H^2\right) + (1-\alpha)\left(\frac{1}{2}(1-w_L^{sb})w_L^{sb}\theta_L^2 - U_L(0,w_H) + U_L(0,w_L^{sb})\right) - (\widehat{U}_H^B - U_H(0,w_H)), \quad (13)$$

subject to *(PCL)*, which we write as $\Delta\widehat{U}^B = \widehat{U}_H^B - \widehat{U}_L^B \geq U_H(0,w_H) - U_L(0,w_H)$.

Observe first that $w^* := w_H^*$ is non-decreasing in $\Delta\widehat{U}_B$ as higher $\Delta\widehat{U}_B$ relaxes *(PCL)*, and as $\frac{\partial}{\partial w}[U_H(0,w) - U_L(0,w)] = \frac{w}{2}(\theta_H^2 - \theta_L^2) > 0$. This implies that w^* is strictly increasing in $\Delta\widehat{U}^B$ if *(PCL)* is binding (Region QC(b)).

The partial derivatives are

$$\frac{\partial}{\partial w}\Pi(w) = \frac{1}{2}\left(\alpha\theta_H^2 + w\left((1-2\alpha)\theta_H^2 - (1-\alpha)\theta_L^2 - 2\alpha\rho\sigma^2\right)\right)$$
$$\frac{\partial^2}{\partial w^2}\Pi(w) = \frac{1}{2}\left((1-2\alpha)\theta_H^2 - (1-\alpha)\theta_L^2 - 2\alpha\rho\sigma^2\right).$$

Let w^* solve the FOC $\frac{\partial}{\partial w}\Pi(w^*) = 0$. If $\Delta\widehat{U}^B > U_H(0,w^*) - U_L(0,w^*)$ and $\frac{\partial^2}{\partial w^2}\Pi(w^*) < 0$ (which are the conditions for Region QC(a)), then, because of the binding *(PCH)*, *(PCL)* is fulfilled but non-binding, and $\Pi(w^*)$ is therefore the greatest profit the good firm can derive. Note that $w^* > 0$ if and only if $\frac{\partial^2}{\partial w^2}\Pi(w^*) < 0$. In this case the maximum is global and w^* is constant whenever $\Delta\widehat{U}_B > \frac{\alpha^2\theta_H^4\left(\theta_H^2 - \theta_L^2\right)}{4\left((2\alpha-1)\theta_H^2 + (1-\alpha)\theta_L^2 + 2\alpha\rho\sigma^2\right)^2}$, where the right-hand side corresponds to the threshold where both w^* satisfies the FOC and *(PCL)* is binding.

On the other hand, *(PCL)* is binding whenever $\Delta\widehat{U}_B \leq \frac{\alpha^2\theta_H^4\left(\theta_H^2 - \theta_L^2\right)}{4\left((2\alpha-1)\theta_H^2 + (1-\alpha)\theta_L^2 + 2\alpha\rho\sigma^2\right)^2}$. In this case w^* just solves the binding *(PCL)*.

If $\frac{\partial^2}{\partial w^2}\Pi(w^*) > 0$, then the piece rate that solves the FOC is negative, so that $\Pi(w)$ is strictly increasing on $[0,1]$ and constrained only by the (then binding) *(PCL)* condition, so that this case corresponds to Region QC(b).

Finally, to prove that in Region QC social welfare is decreasing in $\Delta\widehat{U}^B$, observe first that social welfare is given by

$$W(w_H) = \Pi(F_H, w_H, F_L, w_L) + \alpha\, U_H(F_H, w_H) + (1-\alpha)U_L(F_L, w_L)$$
$$= \alpha\left(\frac{1}{2}w_H\theta_H^2 - \frac{1}{4}w_H^2\theta_H^2 - \frac{1}{2}w_H^2\rho\sigma^2\right) + (1-\alpha)\left(\frac{1}{2}w_L\theta_L^2 - \frac{1}{4}w_L^2\theta_L^2 - \frac{1}{2}w_L^2\rho\sigma^2\right),$$

where $w_L = w_L^{sb}$ is constant in Region QC. We thus need to consider the first two derivaties with respect to w_H, which are given by

$$\frac{\partial}{\partial w_H} W(w_H) = \alpha \left(\frac{1}{2}\theta_H^2 - \frac{1}{2} w_H \theta_H^2 - w_H \rho \sigma^2 \right)$$
$$\frac{\partial^2}{\partial w_H^2} W(w_H) = -\alpha \left(\frac{\theta_H^2}{2} + \rho\sigma^2 \right)$$

and $W(w_H)$ is greatest if $w_H = \dfrac{\theta_H^2}{\theta_H^2 + 2\rho\sigma^2} = w_H^{sb}$. Since $w_H^* > w_H^{sb}$ in Region QC, social welfare is decreasing. □

A.2 The least-cost separating allocation is interim efficient

Proof of Proposition 2.

(i). We first show that the low type has an imitation incentive when both types are offered their output and second-best piece rates. In this case, the fixed rates offered by the bad firm are given by $F_i^B(w) = (1-w)\frac{1}{2}w\beta^2\theta_i^2$, $i \in \{H, L\}$, and the expected utilities for the low type, depending on the contract she choses is given as

$$U_L^B(w_H^{B,sb}) = \frac{1}{2}(1 - w_H^{B,sb}) w_H^{B,sb} \beta^2 \theta_H^2 + \frac{w_H^{B,sb^2}}{4}(\beta^2\theta_L^2 - 2\rho\sigma^2) = \frac{\beta^4\theta_H^4(\beta^2\theta_L^2 + 2\rho\sigma^2)}{4(\beta^2\theta_H^2 + 2\rho\sigma^2)^2}$$
$$U_L^B(w_L^{B,sb}) = \frac{1}{2}(1 - w_L^{B,sb}) w_L^{B,sb} \beta^2 \theta_L^2 + \frac{w_L^{B,sb^2}}{4}(\beta^2\theta_L^2 - 2\rho\sigma^2) = \frac{\beta^4\theta_L^4}{4(\beta^2\theta_L^2 + 2\rho\sigma^2)}.$$

This gives

$$\frac{U_L^B(w_H^{B,sb})}{U_L^B(w_L^{B,sb})} = \frac{(\theta_H^2(\beta^2\theta_L^2 + 2\rho\sigma^2))^2}{(\theta_L^2(\beta^2\theta_H^2 + 2\rho\sigma)^2)^2} = \frac{(\beta^2\theta_L^2\theta_H^2 + 2\rho\sigma^2\theta_H^2)^2}{(\beta^2\theta_L^2\theta_H^2 + 2\rho\sigma^2\theta_L^2)^2} \geq 1,$$

and the claim that the low type has an imitation incentive follows.

Next, we show that there exists $\alpha_{LCS} \in (0,1)$ such that the LCS is interim efficient if and only if $\alpha \leq \alpha_{LCS}$. As the low type has an imitation incentive when both types are offered their expected output, the bad firm offers a pair of quasi-competitive contracts with $w_L^{B,*} = \dfrac{\beta^2\theta_L^2}{\beta^2\theta_L^2 + 2\rho\sigma^2}$ and with the low type's incentive compatibility constraint *(ICCLB)* binding, so that

$$F_H^B(F_L^B, w_H^B) = U_L^B(F_L^B, w_L^{B,*}) - \frac{{w_H^B}^2}{4}(\beta^2\theta_L^2 - 2\rho\sigma^2).$$

The break-even condition is

$$\Pi^B(F_L^B, w_H^B) = \alpha \left\{ \frac{1}{2}(1 - w_H^B) w_H^B \beta^2 \theta_H^2 - U_L^B(F_L^B, w_L^{B,*}) + \frac{{w_H^B}^2}{4}(\beta^2\theta_L^2 - 2\rho\sigma^2) \right\}$$
$$+ (1-\alpha)\left\{ \frac{1}{2} w_L^{B,*}(1 - w_L^{B,*})\beta^2\theta_L^2 - F_L^B \right\} = 0.$$

With the Implicit Function Theorem we obtain

$$\frac{\partial w_H^B(F_L^B)}{\partial F_L^B} = -\frac{\frac{\partial}{\partial F_L^B}\Pi^B(F_L^B, w_H^B)}{\frac{\partial}{\partial w_H^B}\Pi^B(F_L^B, w_H^B)} = -\frac{2}{\alpha(\beta^2(\theta_H^2(2w_H^B - 1) - \theta_L^2 w_H^B) + 2\rho\sigma^2 w_H^B)}.$$

When increasing the low type's utility by increasing F_L^B, the high type's utility changes according to

$$\begin{aligned}\frac{\partial}{\partial F_L^B} U_H^B(F_H^B(F_L^B), w_H^B(F_L^B)) &= \frac{\partial}{\partial F_L^B}\left\{U_L^B(F_L^B, w_L^B) + \frac{\beta^2 w_H^B(F_L^B)^2}{4}(\theta_H^2 - \theta_L^2)\right\} \\ &= 1 + \frac{w_H^B(F_L^B)\, w_H^{B\prime}(F_L^B)\beta^2}{2}(\theta_H^2 - \theta_L^2) \\ &= 1 - \frac{w_H^B \beta^2}{\alpha(\beta^2(\theta_H^2(2w_H^B - 1) - \theta_L^2 w_H^B) + 2\rho\sigma^2 w_H^B)}(\theta_H^2 - \theta_L^2).\end{aligned}$$

This expression is positive if and only if

$$\alpha \geq \frac{w_H^B \beta^2(\theta_H^2 - \theta_L^2)}{w_H^B \beta^2(\theta_H^2 - \theta_L^2) + \beta^2\theta_H^2(w_H^B - 1) + 2\rho\sigma^2 w_H^B} = \frac{w_H^B \beta^2(\theta_H^2 - \theta_L^2)}{w_H^B \beta^2(\theta_H^2 - \theta_L^2) - 2\frac{dv_H^B(w_H^B)}{dw_H^B}}. \quad (14)$$

In particular, if w_H^B refers to a contract pair where each type receives exactly her output,[16] then (14) implies that the high type's certainty equivalent can be increased by offering her less than her output, provided that the right-hand side of (14) is smaller than 1. This is the case if and only if

$$\beta^2\theta_H^2(w_H^B - 1) + 2\rho\sigma^2 w_H^B > 0.$$

This is equivalent to $w_H^B > \dfrac{\beta^2\theta_H^2}{\beta^2\theta_H^2 + 2\sigma\rho^2} = w_H^{B,sb}$ which holds for all quasi-competitive contract menus.

Summing up, there exists $\alpha < 1$ such that both the high type's and the low type's utility increases when deviating from a contract where each type receives her output, while maintaining the break-even condition.

(ii) and **(iii)**. Note further that (14), with the appropriate w_H^B where each type receives exactly her output, implies that $\frac{\partial}{\partial \beta}\alpha_{LCS} \geq 0$ and $\frac{\partial}{\partial \rho}\alpha_{LCS} \leq 0$. □

---

[16]To derive an explicit expression for w_H^B, note first that when each type receives her output, then

$$\begin{aligned}F_L^B &= \frac{1}{2}(1 - w_L^B)w_L^B\beta^2\theta_L^2 \\ F_H^B &= \frac{1}{2}(1 - w_H^B)w_H^B\beta^2\theta_H^2.\end{aligned}$$

From the binding *(ICCLB)*, we can solve for w_H^B:

$$w_H^B = \frac{\beta^2\theta_H^2 + \sqrt{\beta^4\theta_H^4 - 4U_L^B(F_L^B, w_L^B)(\beta^2(2\theta_H^2 - \theta_L^2) + 2\rho\sigma^2)}}{\beta^2(2\theta_H^2 - \theta_L^2) + 2\rho\sigma^2}$$

Proof of Proposition 3. Suppose the bad firm offers LCS contracts (i.e., both types receive their expected output in the bad firm) and the good firm best responds with a contract menu as specified in Proposition 1. The second assumption is without loss of generality as we will show in Lemma 3 that the good firm hires both types of agents in any pure strategy equilibrium.

Case 1: (PCH) non-binding.

Suppose the good firm offers a contract to the high type such that (PCH) is non-binding. By Proposition 1, this implies that (PCL) is binding. Then, the bad firm has no profitable deviation since LCS is IE and hiring any type of agent requires to offer more than their output. In particular, offering more to the high type without attracting her, does not allow for a deviation that profitably attracts the low type. Hence, if LCS is IE and the good firm best responds with (PCH) not binding (i.e., a quasi-monopsonistic contract), there exists a pure strategy equilibrium.

Case 2: (PCH) binding.

Let the good firm offer a contract to the high type such that (PCH) is binding. Then, (PCL) for the good firm's contract offer has to be binding as well. To see this suppose that (PCL) is not binding. Then, the bad firm could increase the utility offered to the low type without attracting her such that the inefficiently high piece rate for the high type could be reduced. This generates more surplus and the high type could be profitably attracted by the bad firm which resembles a profitable deviation. But if (PCL) and (PCH) are binding, the bad firm has no profitable deviation since LCS is IE and hiring any type of agent requires to offer more than their output. Hence, if LCS is IE and the good firm best responds with (PCH) and (PCL) binding, there exists a pure strategy equilibrium. It remains to show that the best response according to Proposition 1 indeed yields a binding (PCL) – i.e., the best response is not in region QC(a). To see this suppose hypothetically that the *good* firm offers both types of agents their output. If LCS is IE in the bad firm, then this particular allocation is also IE because α_{LCS} is increasing in β (see Proposition 2(ii)). Hence, the good firm does not benefit from increasing the low type's rent (and reducing the high type's piece rate distortion) when offering the output to both types of agents. So *if* offering both types their output is a best response for the good firm, (PCL) is binding and the offer is in region QC(b) of Proposition 1. When offering both types of agents their output, $\Delta U^B = U_H^B - U_L^B$ is non-decreasing in β (see Proposition 4). So $\Delta\widehat{U}^B$ for $\beta < 1$ is bounded from above by $\Delta\widehat{U}^B$ for $\beta = 1$. By Proposition 1, $\Delta\widehat{U}^B$ is larger in region QC(a) than in region QC(b). We already saw that the best response against $\Delta\widehat{U}^B$ for $\beta = 1$ (which is offering both types their output) is in region QC(b). Therefore best responses against $\Delta\widehat{U}^B$ for $\beta < 1$ can never be in region QC(a) and (PCL) is indeed binding. □

Proof of Proposition 4. As the bad firm offers a quasi-competitive contract, the incentive compatibility constraint of the low type in the bad firm is binding, i.e., $U_L^B = U_H^B - \frac{(w_H^B)^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$. Hence, $\Delta\widehat{U}^B$ is increasing in β for a given w_H^B and increasing in

w_H^B. The optimal choice of w_H^B equilibrates the marginal loss due to additional rents for low types (i.e., $(1-\alpha)\frac{(w_H^B)^2}{4}\beta^2(\theta_H^2-\theta_L^2)))$ and the marginal gain from reducing piece-rate distortions for high types (i.e., $-\alpha\frac{dv_H^B(w_H^B)}{dw_H^B}$). Taking the first order condition for w_H^B (i.e., $\alpha v_H^{B\prime}(w_H^B)+(1-\alpha)\frac{w_H}{2}(\theta_H^2-\theta_L^2)=0$) as an implicit function, we get $\frac{dw_H^B}{d\beta}>0$ if $\alpha<\alpha_{LCS}$. It follows that $\frac{d\Delta\widehat{U}^B}{d\beta}>0$.

The second statement follows in a similar way. □

A.3 The least-cost separating allocation is *not* interim efficient

Proof of Lemma 2.

(i). We first show that the bad firm offers the high type exactly her expected output. This consists of two parts: First, if she is offered less and *(PCH)* is binding, then the bad firm has a profitable deviation. Second, offering more is weakly dominated as the high type has no imitation incentive in the bad firm.

Suppose the bad firm offers contracts $(F_H^{B,*},w_H^{B,*}),(F_L^{B,*},w_L^{B,*})$ where the high type is offered less than her output. Then, since *(PCH)* is binding, the bad firm has the following profitable deviation: It can increase its offer to the high type and profitably attract her. Of course, the low type must not be attracted, and therefore the profitable deviation entails a greater distortion in the contract designed for the high type. Formally, choose $(\overline{F}_H^B,\overline{w}_H^B)$ with $\overline{w}_H^B>w_H^{B,*}$ and

$$\overline{F}_H^B(\overline{w}_H^B)=U_L^B(F_H^{B,*},w_H^{B,*})-U_L^B(0,\overline{w}_H^B)=F_H^{B,*}+\frac{{w_H^{B,*}}^2-{\overline{w}_H^B}^2}{4}(\beta^2\theta_L^2-2\rho\sigma^2).$$

By construction, $U_L^B(\overline{F}_H^B,\overline{w}_H^B)=U_L^B(F_H^{B,*},w_H^{B,*})$, so *(ICCLB)* is satisfied. Furthermore,

$$\begin{aligned}U_H^B(\overline{F}_H^B,\overline{w}_H^B)&-U_H^B(F_H^{B,*},w_H^{B,*})\\&=U_L^B(F_H^{B,*},w_H^{B,*})-U_L^B(0,\overline{w}_H^B)+U_H^B(0,\overline{w}_H^B)-U_H^B(F_H^{B,*},w_H^{B,*})\\&=\frac{{\overline{w}_H^B}^2-{w_H^{B,*}}^2}{4}(\beta^2\theta_H^2-\beta^2\theta_L^2)>0,\end{aligned}$$

which implies that *(ICCHB)* is satisfied, and that the bad firm attracts the high type because *(PCH)* was binding.

It remains to show that there exists $\overline{w}_H^B$ such that the expected profit from the high type, $\Pi_H^B(\overline{F}_H^B,\overline{w}_H^B)$, is positive. But this follows directly by observing that

$$\Pi_H^B(\overline{F}_H^B(\overline{w}_H^B),\overline{w}_H^B)=\alpha\left\{\frac{1}{2}(1-\overline{w}_H^B)\overline{w}_H^B\beta^2\theta_H^2-F_H^{B,*}-\frac{{w_H^{B,*}}^2-{\overline{w}_H^B}^2}{4}(\beta^2\theta_L^2-2\rho\sigma^2)\right\}$$

is continuous in $\overline{w}^B_H$, and that $\Pi^B_H(F^{B,*}_H, w^{B,*}_H) > 0$ as the high type is offered less than her output. Thus, a profitable deviation exists when the bad firm offers the high type less than her expected output.

Next, the only reason to offer the high type more than her expected output would be to reduce her imitation incentive. Otherwise, this is weakly dominated. It is hence sufficient to show that the high type has no imitation incentive when the bad firm offers two second-best contracts. It was already shown in the proof of Proposition 3 that it is the low type who has an imitation incentive when two second-best contracts are offered where each type receives her expected output, which implies that the high type has no incentive to imitate. Let us now loosen the assumption that the low type is offered exactly her expected output. Given the previous result, offering the low type less than her expected output cannot result in an imitation incentive. The only reason to offer the low type more than her output is to reduce her imitation incentive, and to profitably attract the high type with a more attractive contract. Of course, such a contract does not exist when the high type's piece rate is second-best, and then, offering the low type more than her output is weakly dominated.

(ii). We show that there exists a profitable deviation that allows the bad firm to attract the high type: when *(PCL)* is non-binding, the bad firm can increase its offer to the low type without attracting her, which allows to reduce the inefficiency in the piece rate for the high type. Suppose contracts (F^*_H, w^*_H), (F^*_L, w^*_L), $(F^{B,*}_H, w^{B,*}_H)$, $(F^{B,*}_L, w^{B,*}_L)$ with a non-binding *(PCL)*, $U_L(F^*_L, w^*_L) > U^B_L(F^{B,*}_L, w^{B,*}_L)$, are offered. Choose $(\overline{F}^B_L, \overline{w}^B_L), (\overline{F}^B_H, \overline{w}^B_H)$ such that

$$\overline{w}^B_L = 0, \tag{15}$$

$$\overline{F}^B_L = U_L(F^*_L, w^*_L) =: \widehat{U}^G_L, \tag{16}$$

$$\overline{F}^B_H = \widehat{U}^G_L - U^B_L(0, \overline{w}^B_H), \tag{17}$$

$$U^B_H(\overline{F}^B_H, \overline{w}^B_H) = U_H(F^*_H, w^*_H). \tag{18}$$

Equations (15) and (16) imply that *(PCL)* is binding. Because of Equation (18), the contract $(\overline{F}^B_H, \overline{w}^B_H)$ does not attract the high type, but once we have verified that this contract pair fulfills the constraints and that the high type receives less than her expected output, the existence of a profitable deviation follows from part (i).

By construction, *(PCL)* and *(ICCLB)* are binding. *(ICCHB)* is fulfilled since

$$U^B_H(\overline{F}^B_H, \overline{w}^B_H) = \widehat{U}_H > \widehat{U}_L = \overline{F}^B_L,$$

where the first equality follows from the binding *(PCH)*, which follows from the initially non-binding *(PCL)*.

To see that the high type receives less than her expected output, we first show that $w^{B,*}_H > \overline{w}^B_H$: From *(ICCLB)* in the initial contract, we have

$$U^B_H(F^{B,*}_H, w^{B,*}_H) \leq U^B_L(F^{B,*}_L, w^{B,*}_L) + \frac{\beta^2 {w^{B,*}_H}^2}{4}(\theta^2_H - \theta^2_L).$$

Furthermore,

$$U_H^B(F_H^{B,*}, w_H^{B,*}) = U_H^B(\overline{F}_H^B, \overline{w}_H^B) = \widehat{U}_L^G + \frac{\beta^2 \overline{w}_H^{B\,2}}{4}(\theta_H^2 - \theta_L^2),$$

so that

$$\frac{\beta^2}{4}(w_H^{B,*\,2} - \overline{w}_H^{B\,2})(\theta_H^2 - \theta_L^2) \geq \widehat{U}_L^G - U_L^B(F_L^{B,*}, w_L^{B,*}) > 0,$$

which implies that $w_H^{B,*} > \overline{w}_H^B$.

Next, because the contract offer by the bad firm is quasi-competitive, it follows that $\overline{w}_H^B > w_H^{B,sb}$. Together with $w_H^{B,*} > \overline{w}_H^B$, it follows that the high type receives less than her expected output: Since $w_H^{B,*} > \overline{w}_H^B > w_H^{B,sb}$, her certainty equivalent is higher with $\overline{w}_H^B$ compared to $w_H^{B,*}$ if offered her expected output in both cases. And as her certainty equivalent is unchanged by construction via the choice of $\overline{F}_H^B$, the claim follows.

(iii). Given the analysis of part (i) it suffices to observe that, whenever *(PCL)* is binding, offering the low type less than her output and less than the second-best piece rate allows the bad firm to profitably attract the low type. □

Proof of Lemma 3. Note first that, with exogenous reservation levels of utility $\widehat{U}_i$ and without further restrictions, it could of course be profit-maximizing to employ just one agent type. For instance, if the high type's reservation level of utility, $\widehat{U}_H$, were greater than her output with a second-best piece rate, then it would be optimal to employ only the low type. In our model, however, the reservation levels of utility are endogenously derived from the bad firm's offers, and hence bounded from above. We show that, given these upper bounds, the good firm can never increase its profit by hiring just one agent type. We distinguish two cases:

Case 1. (PCL) and (PCH) are both binding.

Suppose first that the good firm hires both types, as assumed in the best response function (6) and that *(PCL)* and *(PCH)* are binding in its best response. Recall that this is the case in Regions QM(b), SB and QC(a). Due to the good firm's productivity advantage, the bad firm can only derive profit from attracting a single type or both types if this is also the case for the good firm. Therefore, the bad firm will not bid more for a type than is profitable for the good firm. When the good firm offers two contracts, then at most one contract is distorted to prevent the other type from imitating. Offering no contract instead of a distorted contract for the respective type foregoes any positive expected profit from this type. All other contracts are second-best, and profits cannot be increased by not offering contracts due to the binding *(PC)s*. Therefore, not placing an offer for any one type cannot be a profitable deviation.

Case 2. (PCH) is non-binding.

The non-trivial case arises in Region QM(a), where *(PCH)* is non-binding, so that the high type receives an information rent if attracted. For this case, we know from text-book models that, whenever the probability of meeting a low type, $1 - \alpha$, is sufficiently small, it may be profitable to hire only the high type in order to save the information rent. We show that this is not the case in our model. The reason is that the high type's information rent depends on α and, in particular, as $1-\alpha$ tends to 0, so does the high type's information rent. As a consequence it turns out that the profits from offering two contracts versus offering only one contract converge as $1-\alpha \to 0$, with offering two contracts being strictly more profitable than offering only one for arbitrary $1-\alpha > 0$.

Formally, in Region QM(a), when offering two contracts we have

$$\begin{aligned} F_L(w_L) &= \widehat{U}_L^B - U_L(0, w_L) && \text{(binding } \textit{(PCL)}\text{)} \\ F_H(w_L) &= U_H(F_L, w_L) - U_H(0, w_H) && \text{(binding } \textit{(ICCH)}\text{)} \\ w_H &= w_H^{sb} \end{aligned}$$

Furthermore, w_L solves the FOC of the good firm's profit function, and is given by

$$w_L = \frac{\theta_L^2}{\theta_L^2 + 2\rho\sigma^2 + \alpha/(1-\alpha)(\theta_H^2 - \theta_L^2)}.$$

Let us analyse the difference in offering two contracts with non-binding *(PCH)* versus offering one contract with binding *(PCH)*, which is given by (assuming that $\Delta\widehat{U}^B$ is small enough so that we are in Region QM(a))

$$\Delta = \Pi(F_H(w_L), w_H, F_L(w_L), w_L) - \alpha\left(\frac{1}{2}(1-w_H)w_H\theta_H^2 - (\widehat{U}_H^B - U_H(0, w_H))\right).$$

Simplifying yields

$$\Delta = \frac{(1-\alpha)^2\theta_L^4}{4(\alpha(\theta_H^2 - \theta_L^2) + (1-\alpha)(\theta_L^2 + 2\rho\sigma^2))} + \alpha\widehat{U}_H^B - \widehat{U}_L^B. \tag{19}$$

When $\beta = 0$, then both $\widehat{U}_H^B = 0$ and $\widehat{U}_L^B = 0$, and only the first term (a positive constant) remains. On the other hand, on the boundary between Regions QM(a) and QM(b), where *(PCH)* becomes binding, Δ is just the profit derived from the low type. In an equilibrium where the bad firm hires the low type, this would be strictly positive, as (i) $\beta < 1$ (when $\beta = 1$ we are in Region QC, as the low type has an imitation incentive) and (ii) the bad firm would pay the low type at most her output (otherwise the bad firm would not want to hire the low type).

It remains to analyse whether $\Delta > 0$ for $\beta > 0$ in Region QM(a). Since the explicit expression for $\widehat{U}_H^B$ is too involved, we find a lower bound for Δ. First, observe that the constant term in Δ (the first term on the RHS of Equation (19)) can be re-written as

$$\begin{aligned} \frac{(1-\alpha)^2\theta_L^4}{4(\alpha(\theta_H^2 - \theta_L^2) + (1-\alpha)(\theta_L^2 + 2\rho\sigma^2))} &= \frac{w_L^2}{4}(\alpha(\theta_H^2 - \theta_L^2) + (1-\alpha)(\theta_L^2 + 2\rho\sigma^2)) \\ &\geq \frac{{w_H^B}^2}{4}\beta^2(\alpha(\theta_H^2 - \theta_L^2) + (1-\alpha)(\theta_L^2 + 2\rho\sigma^2)), \end{aligned} \tag{20}$$

where the inequality follows since $w_L = \beta w_H^B$ when *(PCH)* is binding and $w_L > \beta w_H^B$ when *(PCH)* is non-binding.[17] Inserting the lower bound for the constant term from Equation (20), we obtain after simplifying

$$\Delta \geq \frac{1}{2}(1-\alpha)w_H^B \left(\beta^2(\theta_H^2(w_H^B - 1) - w_H^B(\theta_L^2 + \rho\sigma^2)) + \rho\sigma^2 w_H^B\right) =: \Delta'.$$

At $\beta = 0$, we have $\Delta' = \frac{1}{2}(1-\alpha)\rho\sigma^2 {w_H^B}^2 > 0$, whereas on the boundary between Regions QM(a) and Region QM(b), we have $\Delta' = \Delta \geq 0$. Furthermore, Δ' as a function of β has at most one zero on $[0, 1]$, so that we can conclude that $\Delta' \geq 0$ in Region QM(a). $\square$

Full specification of equilibrium configuration Before we proceed to the proof of Proposition 5, we first discuss the full specification of the equilibrium configuration. We already know that the bad firm offers both types at least their expected output,[18] cross-subsidizes the low type if the LCS is not interim efficient, and needs to offer the low type a utility such that *(PCL)* is binding in the good firm's best response. These requirements jointly set a *lower bound* on the minimum utility the bad firm offers the low type in equilibrium. At the same time, the good firm's willingness to compete sets an *upper bound*. If the upper bound is below the lower bound, there is no pure-strategy equilibrium, while we have multiple equilibria if it is strictly above.

In order to derive these bounds and the range of equilibria, we need to make the bad firm's best response explicit. In equilibrium, the bad firm's best response satisfies

$$U_L^B(F_L^{B,*}, w_L^{B,*}) = \widehat{U}_L^G = \widehat{U}_L^B \tag{21a}$$

$$F_H^{B,*} = \frac{1}{2}(1 - w_H^{B,*})w_H^{B,*}\beta^2\theta_H^2 \tag{21b}$$

$$w_H^{B,*} \text{ such that } U_L^B(F_H^{B,*}, w_H^{B,*}) = \widehat{U}_L^G. \tag{21c}$$

Equation (21a) states that *(PCL)* is binding, $\widehat{U}_L^G = \widehat{U}_L^B$. The expression for $F_H^{B,*}$ in (21b) implies that the high type receives her expected output, and the expression for

[17] When *(PCH)* is binding we have in Region QM, because of the binding *(ICCH)*,

$$\Delta\widehat{U}^B = U_H(F_L, w_L) - U_L(F_L, w_L) = \frac{w_L^2}{4}(\theta_H^2 - \theta_L^2).$$

In the bad firm, we have because of the binding *(ICCLB)*,

$$\Delta\widehat{U}^B = U_H^B(F_H^B, w_H^B) - U_L^B(F_H^B, w_H^B) = \frac{{w_H^B}^2}{4}\beta^2(\theta_H^2 - \theta_L^2),$$

so that in Region QM(b) we have $w_L = w_H^B\beta$. For Equation (20), we have equality on the boundary between Regions QM(a) and QM(b) and strict inequality in Region QM(a).

[18] In the case where *(PCH)* is non-binding (Region QM(a)), such a contract specification is contained in the bad firm's best response set. Since the exact specification of the high type's contract has no effect on the good firm's best response, we take this particular contract specification as given in concrete calculations, but it should be stressed that this has no effect on the good firm's best response, existence of equilibria and impact of competition.

$w_H^{B,*}$ in (21c) says that *(ICCLB)* is binding. Further, the piece rate for the low type is second-best as any other piece rate is weakly dominated.

Summing up, a pure-strategy equilibrium requires that the best responses of the good and the bad firm are simultaneously given by Equation (6) and Equations (21a)–(21c), respectively, and that the good firm's expected profit from each agent is non-negative. We hence need to establish under which conditions the best responses are given by Equation (6) and Equations (21a)–(21c).

For this, we can express an entire equilibrium configuration via one variable, such as $\widehat{U}_L^B$, the utility the low type is offered by the bad firm: Given $\widehat{U}_L^B$, we can determine $\widehat{U}_H^B$ such that the low type does not imitate and such that the high type receives her expected output in the bad firm. The binding *(PCL)* and $\Delta\widehat{U}^B$ then determine the best response by the good firm.

Proof of Proposition 5. We show that, if $\widehat{U}_L^B \in [\underline{U}_L^B, \overline{U}_L^B]$, then the mutual best responses of both firms are given by Equation (6) for the good firm, resp. (21a)–(21c) by the bad firm, and that a profitable deviation exists for at least one firm if $\widehat{U}_L^B \notin [\underline{U}_L^B, \overline{U}_L^B]$.

In the following denote by $\widehat{U}_{L,LCS}^B$ the utility offered to the low type if the bad firms' LCS is IE. In this case, the low type receives her expected output from a contract with a second-best piece rate; formally: $\widehat{U}_{L,LCS}^B = \frac{\left(w_L^{B,sb}\right)^2}{4}\beta^2\theta_L^2$. Note that $\widehat{U}_{L,CS}^B \geq \widehat{U}_{L,LCS}^B$ as a potential cross-subsidy offers the low type more than her expected output.

Consider first the bad firm whose best response is given by (21a)–(21c): Whenever $\widehat{U}_L^G \geq \widehat{U}_{L,LCS}^B$, there is no profitable deviation to attract solely the low type. Thus, the only profitable deviation aims at attracting the high type or both types. We distinguish two cases, depending on whether the bad firm's best response leads to a binding, respectively non-binding, *(PCH)* in the good firm.

Case 1. (PCH) *is binding.*

In this case, the minimum utility to be offered to the high type so that she cannot be profitably attracted regardless of the actions of the good firm is given by $\widehat{U}_{L,CS}^B$. We formalize the minimum certainty equivalent $\widehat{U}_{L,CS}^B$ to be offered to the low type as result of a potential cross-subsidising strategy between the high and low types. Taking into account that in the good firm's best response *(PCH)* is binding, the bad firm may have a profitable deviation by offering the high type less than her output, while offering more to the low type, since increasing the low type's information rent decreases the piece rate offered to the high type, thus reducing the inefficiency.

The highest utility the bad firm can offer to the high type without incurring a loss (in expectation) when attracting her or both types is given by

$$\widehat{U}_{H,CS}^B := \max_{F_H^B, w_H^B, F_L^B, w_L^B} U_H^B(F_H^B, w_H^B) \tag{22}$$

subject to *(ICCLB), (ICCHB)* and

- $\alpha\left(\frac{1}{2}(1-w_H^B)w_H^B\beta^2\theta_H^2 - F_H^B\right)$
$$+ (1-\alpha)\left(\frac{1}{2}(1-w_L^B)w_L^B\beta^2\theta_L^2 - F_L^B\right) = 0 \quad \text{(break-even)},$$
- $U_L(F_L^B, w_L^B) \geq \widehat{U}_{L,LCS}^B$ (output L).

Any offer by the good firm to the high type with $\widehat{U}_H < \widehat{U}_{H,CS}^B$ will be outbid by the bad firm, so that $\widehat{U}_{H,CS}^B$ is the smallest certainty equivalent to be offered to the high type. The explicit expression for $\widehat{U}_{H,CS}^B$ given below is derived as follows: Using that *(ICCLB)* is binding, the break-even condition and $w_L^B = w_L^{B,sb}$, the high type's certainty equivalent is a function of her piece rate w_H^B only. Maximizing Equation (22) via the first-order condition yields

$$\widehat{U}_{H,CS}^B = \frac{\alpha^2\beta^4\theta_H^4}{4(\beta^2((2\alpha-1)\theta_H^2 + (1-\alpha)\theta_L^2) + 2\alpha\rho\sigma^2)} + \frac{(1-\alpha)\beta^4\theta_L^4}{4(\beta^2\theta_L^2 + 2\rho\sigma^2)}. \tag{23}$$

This needs to be translated into the smallest certainty equivalent to be offered to the low type, which is given by

$$\widehat{U}_{L,CS}^B = U_L^B(F_H^B, w_H^B),$$

with $U_H^B(F_H^B, w_H^B) = \widehat{U}_{H,CS}^B$ and $F_H^B = \frac{1}{2}(1-w_H^B)w_H^B\beta^2\theta_H^2$. This offer ensures that the high type is offered $\widehat{U}_{H,CS}^B$ while fulfilling Equations (21a)–(21c). The explicit expression for $\widehat{U}_{L,CS}^B$ is then given by

$$\widehat{U}_{L,CS}^B = \widehat{U}_{H,CS}^B - \frac{\beta^2\left(\beta^2\theta_H^2 + \sqrt{\beta^4\theta_H^4 - 4\widehat{U}_{H,CS}^B(\beta^2\theta_H^2 + 2\rho\sigma^2)}\right)^2}{4(\beta^2\theta_H^2 + 2\rho\sigma^2)^2}(\theta_H^2 - \theta_L^2),$$

if condition (output L) of the optimisation problem (22) is fulfilled. Note that if (output L) is binding, then the bad firm's LCS is IE and $\widehat{U}_{L,CS}^B = \widehat{U}_{L,LCS}^B$.

Recall next from Lemma 2, that a non-binding *(PCL)* entails a profitable deviation for the bad firm. If *(PCL)* is binding for certainty equivalent $\widehat{U}_L^G$, then it is also binding for all greater certainty equivalents. To see this, observe that it follows directly from Equations (21a)–(21c) that $w_L^{B,*} = w_L^{B,sb}$ and $F_L^{B,*} = \widehat{U}_L^G - U_L^B(0, w_L^{B,sb})$. The explicit solution for the high type's piece rate in the bad firm's best response is then given by[19]

$$w_H^{B,*} = \frac{\beta^2\theta_H^2 + \sqrt{\beta^4\theta_H^4 - 4\widehat{U}_L^G(2\beta^2\theta_H^2 - \beta^2\theta_L^2 + 2\rho\sigma^2)}}{2\beta^2\theta_H^2 - \beta^2\theta_L^2 + 2\rho\sigma^2}. \tag{24}$$

[19]Existence of a solution in $\mathbb{R}$ follows from $w_H^{B,*} \geq w_H^{B,sb}$, and since the right-hand side of Equation (24) is decreasing in $\widehat{U}_L^G$ and smaller than $w_H^{B,sb}$ when the expression in the square root is 0.

Hence, (24) implies $\frac{\partial w_H^B}{\partial \widehat{U}_L^G} \leq 0$, which implies that $\Delta\widehat{U}^B = \frac{{w_H^B}^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$ decreases in $\widehat{U}_L^G$. Furthermore, we know from Proposition 1 that *(PCL)* remains binding as U_L^G increases. Hence, the bad firm's best response to $\widehat{U}_L^G > \max(\widehat{U}_{L,CS}^B, \widehat{U}_{L,PCL}^B)$ does not allow her to profitably attract any type.

The explicit expression for $\widehat{U}_{L,PCL}^B$ is determined by the threshold that separates Regions QC(a) and QC(b) in Proposition 1, that is, where

$$\Delta\widehat{U}^B = \frac{\alpha^2\theta_H^4\left(\theta_H^2 - \theta_L^2\right)}{4\left((2\alpha-1)\theta_H^2 + (1-\alpha)\theta_L^2 + 2\alpha\rho\sigma^2\right)^2}.$$

By the binding *(ICCLB)* constraint we have that

$$\Delta\widehat{U}^B = U_H^B(F_H^B, w_H^B) - U_L^B(F_H^B, w_H^B) = \frac{{w_H^B}^2}{4}\beta^2(\theta_H^2 - \theta_L^2),$$

so that

$$w_H^B = \frac{\alpha\theta_H^2}{\beta((2\alpha-1)\theta_H^2 + (1-\alpha)\theta_L^2 + 2\alpha\rho\sigma^2)}. \tag{25}$$

The resulting low type's certainty equivalent is

$$\widehat{U}_{L,PCL}^B = \frac{1}{2}(1-w_H^B)w_H^B\beta^2\theta_H^2 + \frac{{w_H^B}^2}{4}(\beta^2\theta_L^2 - 2\rho\sigma^2), \tag{26}$$

with w_H^B given by Equation (25).

Case 2. (PCH) *is non-binding.*

In this case, $\max(\widehat{U}_{L,CS}^B, \widehat{U}_{L,PCL}^B) \neq \widehat{U}_{L,PCL}^B$, since at least one of the participation contraints is binding in the good firm's best response. If the maximum is $\widehat{U}_{L,CS}^B > \widehat{U}_{L,LCS}^B$ (i.e., LCS is not IE), then $\widehat{U}_L^G < \widehat{U}_{L,CS}^B$ may hold in equilibrium as it may not give rise to a profitable deviation which attracts the high type: $\widehat{U}_L^G$ can be lowered to the point where either the low type receives her expected output, or where the high type can be profitably attracted, which then requires that *(PCH)* is binding.

Summing up so far, any $\widehat{U}_L^G \geq \underline{U}_L^B$ rules out that the bad firm can profitably attract any type, whereas any $\widehat{U}_L^G < \underline{U}_L^B$ entails that the bad firm can profitably attract at least one type. Since the good firm attracts both types in equilibrium, the latter case does not constitute an equilibrium, while the former case can constitute an equilibrium provided the good firm is willing to bid and provided that best responses are not weakly dominated in the sense of Assumption 1.

Therefore, consider now the good firm's best response as given by Equation (6). The good firm's best response to $\widehat{U}_L^B > \widehat{U}_{L,\max}^B$, where the bad firm offers the low type even more than her expected output in the *good* firm, is to not bid for the low type. Thus, $\widehat{U}_L^B > \widehat{U}_{L,\max}^B$ cannot hold in equilibrium, as we know from Lemma 3 that the good firm hires both types in equilibrium. For $\widehat{U}_L^B \leq \widehat{U}_{L,\max}^B$ and given the binding *(PCL)*, the good firm attracts the low type, for it will otherwise just forego the profit derived from her.

Similarly, the good firm will not offer the high type more than her expected output. Given that the contract for the high type in the bad firm is inefficient and offers exactly her output, this case is subsumed by $\widehat{U}_{L,\max}^B$. This proves that the good firm's best response to any offer below $\widehat{U}_{L,\max}^B$ is to attract both types.

The greatest certainty equivalent offered to the low type such that the high type receives her output is determined by the case when both $w_H^B = w_H^{B,sb}$ and *(ICCLB)* is binding, which is given by

$$\widehat{U}_{L,\text{no imi.}}^B := U_L^B(F_H^{B,sb}, w_H^{B,sb}) = \frac{\beta^4\theta_H^4(\beta^2\theta_L^2 + 2\rho\sigma^2)}{4(\beta^2\theta_H^2 + 2\rho\sigma^2)^2}. \tag{27}$$

As Assumption 1 excludes weakly dominated strategies, the bad firm will not offer the low type more than $\widehat{U}_{L,\text{no imi.}}^B$ if $\widehat{U}_{L,\text{no imi.}}^B > \widehat{U}_{L,LCS}^B$, as the only reason to offer the low type more than her expected output is to keep her from imitating. And as a non-binding *(ICCLB)* implies that this is not necessary, this is weakly dominated.

Finally, the offers by the bad firm must be such that the high type is offered her expected output by the bad firm, cf. Lemma 2. The highest certainty equivalent offered to the *low* type such that the high type receives her expected output fulfills the first-order condition

$$\frac{\partial}{\partial w_H^B}\left[\frac{1}{2}(1-w_H^B)w_H^B\beta^2\theta_H^2 + \frac{{w_H^B}^2}{2}\left(\frac{\beta^2\theta_L^2}{2} - 2\rho\sigma^2\right)\right] = 0,$$

cf. Equations (21b) and (21c), which in turn yields $U_L^B(F_H^B, w_H^B) = \dfrac{\beta^4\theta_H^4}{4(\beta^2(2\theta_H^2 - \theta_L^2) + 2\rho\sigma^2)}$. It is easily shown that this expression is greater than $\widehat{U}_{L,\text{no imi.}}^B$, so that this case can be ignored.

Summing up, $\widehat{U}_L^G < \underline{U}_L^B$ implies that the bad firm has a profitable deviation, whereas $\widehat{U}_L^B > \overline{U}_L^B$ implies that either the good firm does not attract both types or that the bad firm's best response is weakly dominated. Any $\widehat{U}_L^B \in [\underline{U}_L^B, \overline{U}_L^B]$ fulfills the necessary conditions of Lemmas 2 and 3, and is also sufficient as the offered contracts are mutual best responses. □

Proof of Proposition 6. We show that $\dfrac{\partial\Delta\widehat{U}^B}{\partial\beta} \geq 0$, for both $\widehat{U}_L^B = \underline{U}_L^B$ and $\widehat{U}_L^B = \overline{U}_L^B$ in $\Delta\widehat{U}^B$. By Equations (21a)-(21c) we have

$$\Delta\widehat{U}^B(\beta) = U_H^B(0, w_H^{B,*}(\beta)) - U_L^B(0, w_H^{B,*}(\beta)) = \frac{w_H^{B,*}(\beta)^2}{4}\beta^2(\theta_H^2 - \theta_L^2),$$

with $w_H^{B,*}(\beta)$ given according to Equation (24) (observe that $\widehat{U}_L^G$ depends on β). Hence,

$$\frac{\partial}{\partial\beta}\Delta\widehat{U}^B = \frac{w_H^{B,*}(\beta)}{2}\left\{\beta w_H^{B,*\prime}(\beta) + w_H^{B,*}(\beta)\right\}\beta(\theta_H^2 - \theta_L^2),$$

and it is sufficient to show that $w_H^{B,*\prime}(\beta) \geq 0$.

Case 1: $U_L^B = \underline{U}_L^B$.

Since *(PCH)* is binding, $\underline{U}_L^B = \max(\widehat{U}_{L,CS}^B, \widehat{U}_{L,PCL}^B)$. Suppose first that cases do not change at β, and consider the following three cases: $\underline{U}_L^B = \widehat{U}_{L,CS}^B = \widehat{U}_{L,LCS}^B$, $\underline{U}_L^B = \widehat{U}_{L,CS}^B > \widehat{U}_{L,LCS}^B$ and $\underline{U}_L^B = \widehat{U}_{L,PCL}^B$ in turn.

Assume first that $\underline{U}_L^B = \widehat{U}_{L,LCS}^B = \dfrac{\beta^4\theta_L^4}{4(\beta^2\theta_L^2 + 2\rho\sigma^2)}$. Inserting this expression into Equation (24) and taking the derivative, we obtain

$$\begin{aligned} w_H^{B,*\prime}(\beta) = &\frac{4\beta\rho\sigma^2\left(\beta^6(\theta_H^2\theta_L^6 - \theta_H^4\theta_L^4) + \beta^2(\theta_H^2\theta_L^2\sqrt{\beta^8\theta_L^4(\theta_H^2 - \theta_L^2)^2 + 2\rho\sigma^2(\text{pos. terms})}\right)}{\text{positive terms}} \\ &+ \text{positive terms} \\ &\geq 0. \end{aligned}$$

If $\underline{U}_L^B = \widehat{U}_{L,PCL}^B$, by definition $\dfrac{\partial}{\partial\beta}\Delta\widehat{U}^B = 0$.

Last, consider the case where $\underline{U}_L^B = \widehat{U}_{L,CS}^B$, assuming that $\widehat{U}_{L,CS}^B > \widehat{U}_{L,LCS}^B$. Let $\widehat{U}_{H,CS}^B$ be the minimum utility to be offered to the high type by the bad firm from the cross-subsidy strategy where the low type receives some of the high type's expected output, see the proof of Proposition 5, in particular, Equation (22). By definition, we have $U_H^B(F_H^B(w_H^{B,*}), w_H^{B,*}) = \widehat{U}_{H,CS}^B$, and get

$$\frac{\partial}{\partial\beta}U_H^B = \frac{\partial}{\partial\beta}\widehat{U}_{H,CS}^B. \tag{28}$$

To deduce that $w_H^{B,*} \geq 0$ requires making each side of Equation (28) explicit. In the cross-subsidy contract from Equation (22), $w_L^{B,*} = w_L^{B,sb}$ and *(ICCLB)* is binding. Then, the problem that solves the cross-subsidy contract reduces to one variable, the optimal piece rate for the high type, denoted by w^*. As w^* solves the first-order condition $\dfrac{\partial}{\partial w}\,\widehat{U}_{H,CS}^B(w^*, \beta) = 0$, we get

$$\widehat{U}_{H,CS}^B{}'(w^*(\beta), \beta) = \frac{\partial}{\partial\beta}\widehat{U}_{H,CS}^B(w^*, \beta) + w^{*\prime}(\beta)\underbrace{\frac{\partial}{\partial w}\widehat{U}_{H,CS}^B(w^*, \beta)}_{=0},$$

and

$$
\begin{aligned}
\frac{\partial}{\partial\beta}\widehat{U}^B_{H,CS}(w^*,\beta) &= \alpha(1-w^*)w^*\beta\theta_H^2 \\
&\quad + (1-\alpha)\left\{(1-w_L^{B,sb})w_L^{B,sb}\beta\theta_L^2 - \frac{w^{*2}-w_L^{B,sb2}}{2}\beta\theta_L^2\right\} + \frac{w^{*2}}{2}\beta\theta_H^2 \\
&= \frac{2}{\beta}\left\{\widehat{U}^B_{H,CS} - (1-\alpha)\frac{w^{*2}-w_L^{B,sb2}}{2}\rho\sigma^2 + \frac{w^{*2}}{2}\rho\sigma^2\right\} \qquad (29)\\
&= \frac{2}{\beta}\left\{\widehat{U}^B_{H,CS} + (1-\alpha)\frac{w_L^{B,sb2}}{2}\rho\sigma^2 + \alpha\frac{w^{*2}}{2}\rho\sigma^2\right\}
\end{aligned}
$$

On the other hand, with the piece rate offered in equilibrium,

$$
\begin{aligned}
\frac{\partial}{\partial\beta}U_H^B(w_H^{B,*}(\beta),\beta) &= \frac{\partial}{\partial\beta}U_H^B(w_H^{B,*},\beta) + w_H^{B,*\prime}(\beta)\frac{\partial}{\partial w}U_H^B(w_H^{B,*},\beta) \\
&= \frac{2}{\beta}\left\{U_H^B + \frac{w_H^{B,*2}}{2}\rho\sigma^2\right\} + w_H^{B,*\prime}(\beta)\left\{\frac{\beta^2\theta_H^2}{2} - \frac{w_H^{B,*}\beta^2\theta_H^2}{2} - w_H^{B,*}\rho\sigma^2\right\}. \qquad (30)
\end{aligned}
$$

To show that $w_H^{B,*\prime}(\beta) \geq 0$, recall first that because of Equation (28), the expressions (29) and (30) must be identical. Observe further that $\frac{\partial}{\partial w}U_H^B(w,\beta) \leq 0$ for $w \geq w_H^{B,sb}$, the high type's optimal contract when she receives her output. This implies that the second term of Equation (30) is negative if $w_H^{B,*\prime}$ is positive and vice versa. If we show that $w_H^{B,*} > w^*$, which implies that the first term of Equation (30) is greater than Equation (29), then it follows directly by the equality of Equation (29) and Equation (30) that $w_H^{B,*\prime}$ is positive. But to see that $w_H^{B,*} > w^*$ observe that $U_H^B(F_H^B(w^*), w^*) > U^B_{H,CS}$ since the agent receives her full output in the first case, while she receives less in the cross-subsidy case.

It remains to observe that the above results also cover the cases where the case distinction of U_L^B switches due to the continuity of all variables involved.

Case 2: $U_L^B = \overline{U}_L^B$.

We now show that $w_H^{B,*\prime}(\beta) \geq 0$ for $U_L^B = \overline{U}_L^B$. Again, we need to consider the two candidates for $\overline{U}_L^B$ separately. For $\overline{U}_L^B = \widehat{U}_{L,\text{no imi.}}$, by definition $w_H^{B,*} = w_H^{B,sb} = \frac{\beta^2\theta_H^2}{\beta^2\theta_H^2 + 2\rho\sigma^2}$, which is increasing in β. For $\overline{U}_L^B = \widehat{U}_{L,\max}$, we distinguish two cases: First, in Regions 2 and 3, the piece rate for the low type is second-best, so that $\widehat{U}_{L,\max} = \frac{\theta_L^4}{4(\theta_L^2 + 2\rho\sigma^2)}$. In this case, $w_H^{B,*}$ is given by Equation (24) with $\widehat{U}_L^G$ replaced by $\widehat{U}_{L,\max}$, which is a constant that does not depend on β. One can then easily show that $w_H^{B,*}$ is increasing in β. Second, in Region QM(b), the piece rate for the low type is distorted and given by $w_L^* = \beta w_H^{B,*}$, which is easily derived from the expression for w_L^* in Region QM(b) given in Proposition 1 and the binding *(IC-CLB)*. Hence, when the low type receives her full output from the good firm, then $\widehat{U}_L = (1-\beta w_H^{B,*})\beta w_H^{B,*}\frac{\theta_L^2}{2} + \frac{(\beta w_H^{B,*})^2}{4}(\theta_L^2 - 2\rho\sigma^2)$, and plugging this into Equation (24) yields $w_H^{B,*} = \frac{\beta(\beta\theta_H^2 - \theta_L^2)}{\beta^2(\theta_H^2-\theta_L^2)+(1-\beta^2)\rho\sigma^2}$. This expression is non-negative if $\beta\theta_H^2 \geq \theta_L^2$, and, in particular, we have that $w_H^{B,*} \geq w_H^{B,sb} > 0$, because in the bad firm, the contract for the high type is distorted. Hence, it is sufficient to analyze $w_H^{B,*\prime}(\beta)$ under the condition that $\beta\theta_H^2 \geq \theta_L^2$. We have

$$\begin{aligned} w_H^{B,*\prime}(\beta) &= \frac{\beta^2\theta_L^2(\theta_H^2-\theta_L^2)+\rho\sigma^2(2\beta\theta_H^2-\theta_L^2-\beta^2\theta_L^2)}{(\beta^2(\theta_H^2-\theta_L^2-\rho\sigma^2)+\rho\sigma^2)^2} \\ &\geq \frac{\beta^2\theta_L^2(\theta_H^2-\theta_L^2)+\rho\sigma^2(1-\beta^2)\theta_L^2}{(\beta^2(\theta_H^2-\theta_L^2-\rho\sigma^2)+\rho\sigma^2)^2} \geq 0, \end{aligned}$$

and the claim follows.

□

A.4 Including horizontal differentiation

Lemma A.1. *Suppose $0 < x_i^k < 1$ for $i \in \{H, L\}$. Then,*

To enhance readability, we omit the index k of the firm under consideration. As firms only differ in β, no confusion should arise.

1. $w_H^{k,*} \geq w_H^{k,sb}$ *and* $w_L^{k,*} \leq w_L^{k,sb}$;
2. *If $w_L^{k,*} < w_L^{k,sb}$, then: (i) $w_H^{k,*} = w_H^{k,sb}$; (ii) (ICCH) is binding and (iii) (ICCL) is non-binding.*
3. *If $w_H^{k,*} > w_H^{k,sb}$, then: (i) $w_L^{k,*} = w_L^{k,sb}$; (ii) (ICCL) is binding and (iii) (ICCH) is non-binding.*

Proof of Lemma A.1. 1. Consider an interior best response (w_H^*, U_H^*) and (w_L^*, U_L^*) and suppose that $w_H^* < w_H^{sb}$. Let $x_H^* \in (0,1)$ be the location of an H-type agent whose participation constraint is binding, i.e., $x_H^* = \frac{1}{2} + \frac{U_H - \widehat{U}_H}{2t}$. Now consider U_H such that $U_H - x_H^* t = \widehat{U}_H - (1-x_H^*)t$, i.e., (w_H^{sb}, U_H) satisfies PCH for H-type agents located on $[0, x_H^*]$. Observe that ICCH continues to hold as H-type agents (at any given location) receive the same utility under (U_H, w_H^{sb}) and (U_H^*, w_H^*). Recall from the previous section that $v(w_H)$ is increasing in $w_H < w_H^{sb}$. Hence, under (U_H, w_H^{sb}) the same set of agents located in $[0, x_H^*]$ participates and generates a larger profit for the firm (if H-type and only H-type agents choose (U_H, w_H^{sb})). As U_L^* (and thereby x_L^*) is left unaltered, we are left to show that (U_H, w_H^{sb}) and (w_L^*, F_L^*) satisfy ICCL (and L-type agents therefore choose (w_L^*, F_L^*) and receive the same utility as in the original menu of contracts). To see this, observe that the utility for the L type agent who signs the H type's contract is $U_H - \frac{(w_H^{sb})^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$, and

$$U_H - \frac{(\beta w_H^{sb})^2}{4}(\theta_H^2 - \theta_L^2)$$

$$< U_H^* - \frac{(\beta w_H^*)^2}{4}(\theta_H^2 - \theta_L^2)$$

which is the utility enjoyed by an L type agent who signs the original (*) contract (here, we used the definition of U_H (i.e., $U_H = U_H^*$) and $w_H^* < w_H^{sb}$). Hence, imitation incentives are strictly smaller under (w_H^{sb}, F) and ICCL is satisfied whenever it was satisfied in the original menu. This contradicts the optimality of (w_H^*, F_H^*). The proof for $w_L^* \leq w_L^{sb}$ proceeds analogously.

2. Suppose there is an optimal menu of contracts (w_H^*, U_H^*) and (w_L^*, U_L^*) with $w_H^* > w_H^{sb}$. We proceed in three steps. (i) We show that ICCH is not binding whenever ICCL is binding, (ii) we show that ICCL is binding, and (iii) we argue that $w_L^* = w_L^{sb}$.

(i) Suppose that ICCL is binding, i.e., $U_L = U_H - \frac{(w_H^*)^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$. Then, we can rewrite

$$U_H - U_L = \frac{(w_H^*)^2}{4}\beta^2(\theta_H^2 - \theta_L^2) > \frac{(w_L^*)^2}{4}\beta^2(\theta_H^2 - \theta_L^2).$$

The strict inequality follows from $w_H^* > w_H^{sb} > w_L^{sb} \geq w_L^*$ (see Part 1 of the Lemma). Hence, ICCH is not binding whenever ICCL is binding.

(ii) For given U_H and U_L, ICCL reads $U_H - U_L \leq (w_H^*)^2\beta^2(\theta_H^2 - \theta_L^2)$. As $v_H(w_H)$ and thereby Π_H for a given U_H is decreasing in $w_H^* > w_H^{sb}$, it is optimal for the firm to choose the minimal w_H that satisfies ICCH and ICCL. But as ICCH is automatically satisfied whenever ICCL is binding, an optimal w_H^* will be such that ICCL is binding.

(iii) Recall from Part 1 that $w_L^* \leq w_L^{sb}$. As demonstrated in (i) and (ii), ICCL is binding and ICCH is non-binding for any w_L^*. But as $v_L(w_L)$ (and thereby Π_L for a given U_H and U_L) is maximized by w_L^{sb}, it follows that $w_L^* = w_L^{sb}$. □

Proposition A.1. *Suppose* $0 < x_i^k < 1$ *for* $i \in \{H, L\}$ *and* $\alpha < \alpha_{LCS}$, *then*

(i) the best-response utility difference $\Delta U^k = U_H^k - U_L^k$ *is monotone increasing in the utility difference offered by the competitor, i.e.* $\frac{\partial \Delta U^k}{\partial \Delta \widehat{U}^k} > 0$;

(ii) there are utility differences offered by the competitor $\Delta \widehat{U}_{QM}^{\bar{k}}$ *and* $\Delta \widehat{U}_{QC}^{\bar{k}}$ *with* $\Delta \widehat{U}_{QM}^{\bar{k}} < \Delta \widehat{U}_{QC}^{\bar{k}}$ *such that contract offers* $(U_H^k, w_H^k; U_L^k, w_L^k)$ *are*

- quasi-monopsonic *if* $\Delta \widehat{U}^{\bar{k}} < \Delta \widehat{U}_{QM}^{\bar{k}}$.
- second-best *if* $\Delta \widehat{U}_{QM}^{\bar{k}} \leq \Delta \widehat{U}^{\bar{k}} \leq \Delta \widehat{U}_{QC}^{\bar{k}}$
- quasi-competitive *if* $\Delta \widehat{U}^{\bar{k}} > \Delta \widehat{U}_{QC}^{\bar{k}}$

Proof of Proposition A.1. (i) To simplify expressions, we consider U_L and ΔU as a firm's choice variables (again dropping index k if no confusion can arise) rather than U_i and w_i as in (8). As Lemma A.1 indicates that w_i is either second best or w_i^2 is proportional to ΔU this is an equivalent formulation of the firm's optimization program. We prove that $\frac{\partial \Delta U}{\partial \Delta \widehat{U}} > 0$ for quasi-competitive contracts. The proof for quasi-monopsonistic and second-best contracts proceeds analogously. In the same way, we can also demonstrate that $\frac{\partial U_L}{\partial \Delta \widehat{U}} > 0$, $\frac{\partial U_L}{\partial \widehat{U}_L} > 0$, and $\frac{\partial \Delta U}{\partial \widehat{U}_L} > 0$.

For a quasi-competitive contract, $w_L^* = w_L^{sb}$ and $\Delta U = \frac{w_H^2 \beta^2}{4}(\theta_H^2 - \theta_L^2)$ induce the following FOCs for ΔU and U_L:

$$F_{U_L} \equiv \alpha \Pi_H - \alpha(t + U_H - \widehat{U}_H) + (1-\alpha)\Pi_L - (1-\alpha)(t + U_L - \widehat{U}_L) = 0$$

$$F_{\Delta U} \equiv \alpha \Pi_H + \alpha(t + U_H - \widehat{U}_H)(v_H'(w_H)\frac{dw_H}{d\Delta U} - 1) = 0$$

The associated Hessian matrix satisfies $\det(H) > 0$ whenever $(1-\alpha) + \alpha v_H'(w_H)\frac{dw_H}{d\Delta U} \geq 0$ (details available on request). Observe that $(1-\alpha) + \alpha v_H'(w_H)\frac{dw_H}{d\Delta U} = (1-\alpha) + \alpha v_H'(w_H)\frac{1}{\beta^2(\theta_H^2 - \theta_L^2)\frac{w_H}{2}}$. Hence, this condition is satisfied if and only if

$$\alpha v'(w_H) + (1-\alpha)\frac{w_H}{2}\beta^2(\theta_H^2 - \theta_L^2) \geq 0.$$

As indicated by Proposition 2, this condition is satisfied if $\alpha < \alpha_{LCS}$.

By Cramer's rule, $\frac{\partial \Delta U}{\partial \Delta \widehat{U}} = -\frac{\det(H_{\Delta U, \Delta \widehat{U}})}{\det(H)}$ where $H_{\Delta U, \Delta \widehat{U}}$ is the matrix with the first row being $(\frac{\partial F_{U_L}}{\partial U_L}, \frac{\partial F_{U_L}}{\partial \Delta \widehat{U}})$ and the second row being $(\frac{\partial F_{\Delta U}}{\partial U_L}, \frac{\partial F_{\Delta U}}{\partial \Delta \widehat{U}})$. $\det(H_{\Delta U, \Delta \widehat{U}}) < 0$ (such that $\frac{\partial \Delta U}{\partial \Delta \widehat{U}} > 0$) whenever $\alpha v'(w_H) + (1-\alpha)\frac{w_H}{2}\beta^2(\theta_H^2 - \theta_L^2) \geq 0$ (detailed computations available on request).

(ii) For $\Delta \widehat{U} = 0$ and $\widehat{U}_L = \overline{U}$, the best response is the monopsony menu with $\Delta U = \Delta U^m$ and $U_L = U_L^m$. For $\Delta \widehat{U} = \Delta U^c \equiv \frac{(w_H^c)^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$ and $\widehat{U}_L = U_L^c$, the best response is $\Delta U = \Delta U^c > \Delta U^m$ and $U_L = U_L^c > U_L^m$. As the best responses ΔU and U_L are continuous and strictly monotone increasing in $\Delta \widehat{U}$ and $\widehat{U}_L$ (see Part (i)) there is a unique $\Delta \widehat{U}_1$ such that the best response is $\Delta U_1 = \frac{(w_L^{sb})^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$ and a unique $\Delta \widehat{U}_2$ such that the best response is $\Delta U_2 = \frac{(w_H^{sb})^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$. □

Proof of Lemma 4. Consider firm k and denote firm $\bar{k}$'s contract offer by $\widehat{()}$. We proceed as follows. We will first show that both firms will hire some agents as long as β is sufficiently large and second prove that no type of agent will be excluded under these conditions.

1. Suppose $\widehat{()}$ is such that $\widehat{U}_i < \overline{U} + t$ for at least one type $i = H, L$, i.e., agents of type i that are sufficiently close to firm k prefer the outside option to the other firm's offer. As long as $v_i(w_i^{sb}) > \overline{U}$, the firm receives a positive profit from offering $(w_i^{sb}, \overline{U} + \epsilon)$. As $\overline{U}$ is normalized to zero, $v_i(w_i^{sb}) > \overline{U}$ always holds for the good firm and holds for the bad firm on an open neighborhood of $\beta = 1$..

2. Suppose $\widehat{()}$ is such that $\widehat{U}_i \geq \overline{U} + t$ for all types i, i.e., for all agents, the offer of firm $\bar{k}$ resembles the outside option. In equilibrium, firm $\bar{k}$ only offers this menu if it generates positive expected profits. If firm k offers the same menu of contracts, half of the agents of both types will be attracted by firm k. If firm k is the good firm or β is sufficiently close to 1, this also generates a positive profit for firm k. Hence, the good firm and the bad firm for β in an open neighborhood of $\beta = 1$ will always hire a positive mass of agents in any equilibrium. It remains to discuss conditions under which both firms hire agents of both types.

3. For the good firm and for the bad firm on a open neighborhood of $\beta = 1$, there is no incentive to exclude H types in any equilibrium. To see this, suppose firm k does not hire H types but a positive mass of L types and receives positive profits, i.e., $U_L > \widehat{U}_L - t$, $U_L > \overline{U}$, $U_L < v_L(w_L)$, and (by optimality) $w_L = w_L^{sb}$. We have to distinguish three cases. Case 1: Suppose firm $\bar{k}$ offers a quasi-monopsony contract, i.e., $\widehat{U}_H = \widehat{U}_L + \frac{\widehat{w}_L^2}{4}\widehat{\beta}^2(\theta_H^2 - \theta_L^2)$. Then, an H-type agent receives $\widetilde{U}_H = U_L + \frac{(w_L^{sb})^2}{4}\beta^2(\theta_H^2 - \theta_L^2) > \widehat{U}_L - t + \frac{(w_L^{sb})^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$ from accepting the contract offered by firm k to L types. Whenever $\beta w_L^{sb} \geq \widehat{\beta}\widehat{w}_L$, this is at least $\widehat{U}_H - t$ such that a positive mass of H types prefers this contract to the contract menu offered by firm $\bar{k}$ (and by $U_L > \overline{U}$ to the outside option). Hence, a positive mass of H types signs the contract with firm k and H types are not excluded on an open neighborhood of $\beta = 1$ Case 2: Suppose firm $\bar{k}$ offers a quasi-competitive or second-best contract with $\widehat{\Pi}_H > 0$ (which holds whenever $t > 0$) and $U_L > \widehat{U}_L$. Then, firm k generates positive profits from offering $(\widehat{w}_H, \widehat{U}_H)$. To see this, observe that L types prefer (w_L, U_L) as $U_L > \widehat{U}_L \geq \widehat{U}_H - \frac{\widehat{w}_H^2}{4}\widehat{\beta}^2(\theta_H^2 - \theta_L^2)$ where the last inequality follows from ICCL. Whenever $\beta \geq \widehat{\beta}$, this (weakly) exceeds $\widehat{U}_H - \frac{\widehat{w}_H^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$ and there is an open neighborhood of $\beta = 1$ such that $U_L \geq U_H - \frac{w_H^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$ if $1 = \widehat{\beta} > \beta$. Case 3: Suppose firm $\bar{k}$ offers a quasi-competitive or second-best contract with $\widehat{\Pi}_H > 0$ and $U_L \leq \widehat{U}_L$. Then, firm k generates positive profits from offering $(\widehat{w}_H, U_L + \frac{\widehat{w}_H^2}{4}\beta^2(\theta_H^2 - \theta_L^2))$. This contract leaves L types indifferent to (w_L, U_L) as ICCL is binding. But as firm k attracts a positive mass of L types, $U_L > \widehat{U}_L - t$ such that $U_H = U_L + \frac{\widehat{w}_H^2}{4}\beta^2(\theta_H^2 - \theta_L^2) > \widehat{U}_L - t + \frac{\widehat{w}_H^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$ which (weakly) exceeds $\widehat{U}_H - t$ whenever $\beta \geq \widehat{\beta}$ by ICCL in firm $\bar{k}$. Moreover, $U_L > \widehat{U}_L - t$ also implies that there is an open neighborhood of $\beta = 1$ such that $U_H > \widehat{U}_H - t$ for $1 = \widehat{\beta} > \beta$.

4. Finally, for the good firm and for the bad firm on a open neighborhood of $\beta = 1$, there is no incentive to exclude L types in any equilibrium. The incentive to exclude L types is maximal in monopsony (i.e., $t \to \infty$), as the surplus generated by L types is decreasing in t and H types have an imitation incentive in this case.In the monopsony case, firm k maximizes $\Pi = \alpha(v_H(w_H) - U_H) + (1-\alpha)(v_L(w_L) - U_L)$ s.t. the binding ICCH, i.e., $U_H = U_L + \frac{w_L^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$. This yields as a FOC: $(1-\alpha)v_L'(w_L) = \alpha\frac{w_L}{2}\beta^2(\theta_H^2 - \theta_L^2)$. Observe that this implies that the optimal w_L is decreasing in α and $w_L = 0$ for $\alpha = 1$ as $v'(w_L)$ is bounded for $w_L \leq w_L^{sb}$. Excluding L types is optimal if the expected profit with the optimal contract for low types is below the rent savings for

high types, i.e.,

$$(1-\alpha)\Pi_L \leq \alpha \frac{w_L^2}{4}\beta^2(\theta_H^2 - \theta_L^2). \ (*)$$

As Π_L is bounded for $w_L \leq w_L^{sb}$ and $w_L = 0$ for $\alpha = 1$, both sides of $(*)$ are zero for $\alpha = 1$. Now observe that the slope of the LHS (with respect to α) is $-\Pi_L + (1-\alpha)v_L'(w_L)\frac{dw_L}{d\alpha}$ and the slope of the RHS is $\frac{w_L^2}{4}\beta^2(\theta_H^2 - \theta_L^2) + \alpha\frac{w_L}{2}\beta^2(\theta_H^2 - \theta_L^2)\frac{dw_L}{d\alpha}$.Then, the FOC implies that the LHS decreases more steeply in α than the RHS (and coincide at $\alpha = 1$). Hence, $(*)$ is never satisfied as a strict inequality and excluding L types because of rent savings never resembles a strictly better reply. □

Proof of Lemma 5. We need to rule out cases where for at least one type i and at least one firm $U_i > \widehat{U}_i + t$.

Case 1: Suppose $U_i > \widehat{U}_i + t$ for $i = H, L$ in a profit maximizing contract menu. Then, the firm could lower U_i for both types without altering incentive compatibility and individual rationality for both types on all locations. A contradiction to profit maximization.

Case 2: Suppose $U_H - \widehat{U}_H \leq t < U_L - \widehat{U}_L$, i.e., $x_L = 1$ and $x_H \leq 1$. According to Lemma 4, there is an open neighborhood of $\beta = 1$ such that no firm excludes any of the two types. Hence, $\widehat{U}_L > \overline{U}$ such that $U_L > \overline{U}$. The firm's objective is therefore to maximize $\alpha x_H \Pi_H + (1-\alpha)\Pi_L$ subject to $U_H \geq U_L + \frac{w_L^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$ (ICCH, multiplier μ_H) and $U_L \geq U_H - \frac{w_H^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$ (ICCL, multiplier μ_L). Maximizing with respect to U_L yields the first order condition $-(1-\alpha) - \mu_H + \mu_L = 0$. As ICCL is binding in Case 2 (otherwise $U_L > \widehat{U}_L + t$ cannot be optimal), we get $\mu_L > 0$ and $\mu_H = 0$ such that $\mu_L = (1-\alpha)$. Furthermore, the binding ICCL, a satisfied ICCL of the offer by the other firm, and $U_H - \widehat{U}_H \leq t < U_L - \widehat{U}_L$ implies

$$\frac{w_H^2}{4}\beta^2(\theta_H^2 - \theta_L^2) = U_H - U_L < \widehat{U}_H - \widehat{U}_L \leq \frac{\widehat{w}_H^2}{4}\widehat{\beta}^2(\theta_H^2 - \theta_L^2)$$

or $\beta w_H < \widehat{\beta}\widehat{w}_H$. Then, the first order condition for w_H reads

$$\alpha x_H v_H'(w_H) + (1-\alpha)\frac{w_H}{2}\beta^2(\theta_H^2 - \theta_L^2) = 0 \, (*).$$

If LCS for $t = 0$ with piece-rates w_i^c is IE, i.e., $\alpha v'(w_H^c) + \frac{w_H^c}{2}\beta^2(\theta_H^2 - \theta_L^2) \geq 0$, we get $\alpha x_H v'(w_H) + (1-\alpha)\frac{w_H}{2}\beta^2(\theta_H^2 - \theta_L^2) > 0$ for all $w_H < w_H^c$ and $x_H \leq 1$ (see proof of Proposition 2). Hence, cornering is inferior in Case 2.

Case 3: Suppose $U_L - \widehat{U}_L \leq t < U_H - \widehat{U}_H$, i.e., $x_H = 1$ and $x_L \leq 1$. As no type is excluded by the other firm, $\widehat{U}_L > \overline{U}$ such that $U_L > \overline{U}$. The firm's objective is therefore to maximize $\alpha\Pi_H + (1-\alpha)x_L\Pi_L$ subject to $U_H \geq U_L + \frac{w_L^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$ (ICCH, multiplier μ_H) and $U_L \geq U_H - \frac{w_H^2}{4}\beta^2(\theta_H^2 - \theta_L^2)$ (ICCL, multiplier μ_L). Maximizing with respect to U_L yields the first order condition $-\alpha + \mu_H - \mu_L = 0$. As ICCH is binding in Case 3 (otherwise cornering H types would be suboptimal), we get $\mu_H > 0$ and $\mu_L = 0$ such that $\mu_H = \alpha$. Furthermore, the binding ICCH, a satisfied ICCL of the offer by the other firm, and $U_L - \widehat{U}_L \leq t < U_H - \widehat{U}_H$ implies

$$\frac{w_L^2}{4}\beta^2(\theta_H^2 - \theta_L^2) = U_H - U_L > \widehat{U}_H - \widehat{U}_L \geq \frac{\widehat{w}_L^2}{4}\widehat{\beta}^2(\theta_H^2 - \theta_L^2)$$

or $\beta w_L \geq \widehat{\beta}\widehat{w}_L \geq \beta w_L^m$. Then, the first order condition for w_L reads

$$(1-\alpha)x_L v_L'(w_L) - \alpha\frac{w_L}{2}\beta^2(\theta_H^2 - \theta_L^2) = 0\,(*).$$

In the monopsony case (see above), the corresponding first order condition reads $(1-\alpha)v_L'^m) = \alpha\frac{w_L^m}{2}\beta^2(\theta_H^2 - \theta_L^2)$. Inserting the first order condition for monopsony into $(*)$ yields $x_L = \frac{v'(w_L^m)}{v'(w_L)}\frac{w_L}{w_L^m} > 1$. As $w_L \geq w_L^m$ we get a contradiction. Hence, cornering H types is never optimal in Case 3 which concludes the proof. □

Proof of Proposition 7. We proceed in two steps: First, we show that if cornering is not feasible (i.e., $U_i^k \leq U_i^{\bar{k}} + t$) and β is sufficiently large to admit an interior solution, there is always a pure strategy equilibrium C^*. I.e., C^* is an equilibrium if and only if cornering is not a profitable deviation. Second, we demonstrate that cornering is not a best response against C^* if and only if C^* is interim efficient, i.e., if firm $\bar{k}$ offers the menu in C^*, offering C^* is Pareto efficient.

Step 1: If cornering is not feasible, best responses U_L^k and ΔU^k are continuous and strictly monotone increasing functions of $\widehat{U}_L^k$ and $\Delta\widehat{U}^k$ (see Proposition A.1). As the strategy space is a lattice, this implies by Milgrom and Roberts (1994, Theorem 3) the existence of a pure strategy equilibrium C^*. It remains to show that if β is sufficiently large to render exclusion unprofitable (see Lemma 4), C^* constitute an equilibrium if and only if cornering is not a profitable deviation against C^*.

$\Leftarrow$ If cornering is not a profitable deviation against C^*, C^* remains an equilibrium if cornering is feasible.

$\Rightarrow$ If cornering is a profitable deviation against C^*, C^* is not a pure strategy equilibrium if cornering is feasible.

But if mutual best replies without cornering do not constitute a pure strategy equilibrium as soon as cornering is permitted, any pure strategy equilibrium of the game with cornering being feasible has to involve a cornering contract. But a cornering contract can never be part of a pure strategy equilibrium. To see this recall that cornering H types is never optimal (see the proof of Lemma 5) and cornering L types is only optimal for firm k if it offers a quasi-competitive contract and cornering allows to reduce imitation incentives for L types to reduce piece rate distortions for H types. Now suppose firm $\bar{k}$ attracts all L types, i.e., $U_L \leq \widehat{U}_L - t$, and firm k attracts some H types, i.e., $U_H > \widehat{U}_H - t$. Then, $U_H - U_L > \widehat{U}_H - \widehat{U}_L$. As firm $\bar{k}$ offers a quasi-competitive contract (see above), ICCL is binding and $\widehat{U}_H - \widehat{U}_L = \frac{\widehat{w}_H^2}{4}\widehat{\beta}^2(\theta_H^2 - \theta_L^2)$. Then, ICCL for contracts offered by firm k implies $\beta w_H > \widehat{\beta}\widehat{w}_H$. If $1 = \widehat{\beta} \geq \beta$, this implies $w_H > \widehat{w}_H$ and the gain from cornering $-\alpha v'(w_H) - (1-\alpha)\frac{w_H}{2}\beta^2(\theta_H^2 - \theta_L^2)$ (which is decreasing in β and increasing in w_H) is larger for firm k than for firm $\bar{k}$. This result is unaltered if $1 = \beta \geq \widehat{\beta}$ and $\widehat{\beta}$ is sufficiently close to 1. Hence, if the good firm (or the bad firm for β sufficiently close to 1) corners, the other firm has an incentive to not exclude L types. Then, cornering cannot be part of an equilibrium strategy and the sub-optimality of cornering against C^* is necessary and sufficient for the existence of a pure strategy equilibrium if β is sufficiently close to 1.

Step 2: We are left to show that cornering against C^* is not a profitable deviation if and only if C^* is interim efficient. $\Leftarrow$ As in the proof of Lemma 5, if C^* is interim efficient, a cornering strategy cannot generate more surplus than C^* and cannot constitute a profitable deviation.

$\Rightarrow$ If cornering is a profitable deviation against C^*, (9) does not hold for w_H^* and C^* is not a best response against C^*. □

Proposition A.2. *For $t > 0$ and $\alpha < \alpha^*_{LCS}$ there is $\beta_t < 1$ such that for all $\beta_t < \beta < 1$, the pure-strategy equilibrium C^* satisfies $\frac{\partial \Delta U^{k,*}}{\partial t} < 0$ and $\frac{\partial \Delta U^{k,*}}{\partial \beta} > 0$.*

Proof of Proposition A.2. According Milgrom and Roberts (1994, Theorem 3) it suffices to show that best responses are strictly monotone decreasing in t and strictly monotone increasing in β. This can be established for interim efficient C^* by routine computations using Cramer's rule as in the proof of Proposition A.1. Consider, e.g., $\frac{d\Delta U}{dt}$ and recall from the proof of Proposition A.1 the first order conditions for ΔU and U_L:

$$F_{U_L} \equiv \alpha \Pi_H - \alpha(t + U_H - \widehat{U}_H) + (1-\alpha)\Pi_L - (1-\alpha)(t + U_L - \widehat{U}_L) = 0$$

$$F_{\Delta U} \equiv \alpha \Pi_H + \alpha(t + U_H - \widehat{U}_H)(v_H'(w_H)\frac{dw_H}{d\Delta U} - 1) = 0$$

By Cramer's rule $\frac{d\Delta U}{dt} = -\frac{\det(H_{\Delta U,t})}{\det(H)}$ where $H_{\Delta U,t}$ is the matrix with the first row being $(\frac{\partial F_{U_L}}{\partial U_L}, \frac{\partial F_{U_L}}{\partial t})$ and the second row being $(\frac{\partial F_{\Delta U}}{\partial U_L}, \frac{\partial F_{\Delta U}}{\partial t})$. If C^* is interim efficient, i.e., $(1-\alpha)x_H^* v'(w_H^*) + (1-\alpha)\frac{w_H^*}{2}\beta^2(\theta_H^2 - \theta_L^2) \geq 0$, it follows that $\det(H_{\Delta U,t}) > 0$ (such that $\frac{d\Delta U}{dt} < 0$). □

References

Acharya, V., M. Pagano, and P. Volpin (2016): "Seeking Alpha: Excess Risk Taking and Competition for Managerial Talent," *Review of Financial Studies*, 29(10), 2565â2599.

Arnott, R., and J. Stiglitz (1991): "Equilibrium in competitive insurance markets with moral hazard," NBER Working Paper No. 3588.

Attar, A., T. Mariotti, and F. Salanié (2014a): "Multiple contracting in insurance markets: Cross-subsidies and quantity discounts," Toulouse School of Economics, Working Paper No IDEI-839.

——— (2014b): "Nonexclusive competition under adverse selection," *Theoretical Economics*, 9, 1–40.

Bannier, C., E. Feess, and N. Packham (2013): "Competition, Bonuses, and Risk-taking in the Banking Industry," *Review of Finance*, 17, 653–690.

Baumol, W., and E. Bailey (1984): "Deregulation and the Theory of Contestable Markets," *Yale Journal on Regulation*, 1, 111–137.

Baumol, W., J. Panzar, and R. Willig (1982): *Contestable Markets and the Theory of Industry Structure*. Harcourt College Publishing.

Bénabou, R., and J. Tirole (2016): "Bonus culture: Competitive pay, screening and multitasking," *Journal of Political Economy*, 124, 305–370.

Bisin, A., and P. Gottardi (1999): "Competitive equilibria with asymmetric information," *Journal of Economic Theory*, 87, 1–48.

Dasgupta, P., and E. Maskin (1986): "The existence of equilibrium in discontinuous economic games, ii: Applications," *The Review of Economic Studies*, 53, 27–41.

Garrett, D., R. Gomes, and L. Maestri (2019): "Competitive screening under heterogeneous information," *Review of Economic Studies*, 86, 1590–1630.

Handel, B., I. Hendel, and M. D. Whinston (2015): "Equilibria in Health Exchanges: Adverse Selection Versus Reclassification Risk," *Econometrica*, 83(4), 1261–1313.

Haushalter, D., S. Klasa, and W. Maxwell (2007): "The influence of product market dynamics on a firmâs cash holdings and hedging behavior," *Journal of Financial Economics*, 84, 797–825.

Hellwig, M. (1987): "Some Recent Developments in the Theory of Competition in Markets with Adverse Selection," *European Economic Review*, 31, 319–325.

Hoberg, G., G. Philipps, and N. Prabhala (2014): "Product market threats, payouts, and financial flexibility," *Journal of Finance*, 69(1), 293–324.

Holmstrom, B., and P. Milgrom (1987): "Aggregation and Linearity in the Provision of Intertemporal Incentives," *Econometrica*, 55, 303–328.

Inderst, R., and A. Wambach (2001): "Competitive insurance markets under adverse selection and capacity constraints," *European Economic Review*, 45(10), 1981–1992.

Jullien, B. (2000): "Participation constraints in adverse selection models," *Journal of Economic Theory*, 93, 1–47.

Lewis, T., and D. Sappington (1989): "Countervailing incentives in agency problems," *Journal of Economic Theory*, 49, 294–313.

Maggi, G., and A. Rodriguez-Clare (1995): "On countervailing incentives," *Journal of Economic Theory*, 66, 238–263.

Maskin, E., and J. Tirole (1992): "The Principal-Agent Relationship with an Informed Principal, II: Common Values," *Econometrica*, 60, 1–42.

Milgrom, P., and J. Roberts (1994): "Comparing Equilibria," *American Economic Review*, 84(3), 441–459.

Mimra, W., and A. Wambach (2011): "A game-theoretic foundation for the Wilson equilibrium in competitive insurance markets with adverse selection," CESifo Working Paper No. 3412.

——— (2014): "New Developments in the Theory of Adverse Selection in Competitive Insurance," *The Geneva Risk and Insurance Review*, 39, 136–152.

Miyazaki, H. (1977): "The Rat Race and Internal Labor Markets," *Bell Journal of Economics*, 8, 394–418.

Mussa, M., and S. Rosen (1978): "Monopoly and product quality," *Journal of Economic Theory*, 18, 301–317.

Netzer, N., and F. Scheuer (2014): "A game theoretic foundation of competitive equilibria with adverse selection," *International Economic Review*, 55, 399–422.

Packham, N. (2018): "Optimal contracts under competition when uncertainty from adverse selection and moral hazard are present," *Statistics and Probability Letters*, 137, 99–104.

Picard, P. (2014): "Participating insurance contracts and the RothschildâStiglitz equilibrium puzzle," *The Geneva Risk and Insurance Review*, 39, 153–175.

Pouyet, J., B. Salanié, and F. Salanié (2008): "On competitive equilibria with asymmetric information," *The B.E. Journal of Theoretical Economics*, 8(1), 1–14.

Riley, J. (1979): "Informational Equilibrium," *Econometrica*, 47, 331–359.

——— (2001): "Silver signals: Twenty-five years of screening and signaling," *Journal of Economic Literature*, 39, 432–478.

Rothschild, M., and J. Stiglitz (1976): "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," *Quarterly Journal of Economics*, 90, 629–649.

Salanié, B. (2005): *The Economics of Contracts*. MIT Press, 2 edn.

Schmidt-Mohr, U., and J. Villas-Boas (1999): "Oligopoly with asymmetric information: Differentiation in credit markets," *RAND Journal of Economics*, 30, 375–396.

——— (2008): "Competitive product lines with quality constraints," *Quantitative Marketing and Economics*, 6, 1–16.

Simon, L., and M. Stinchcombe (1995): "Equilibrium refinement for infinite normal-form games," *Econometrica*, 63, 1421–1443.

Spulber, D. (1989): "Product variety and competitive discounts," *Journal of Economic Theory*, 48, 510–525.

Stole, L. (1995): "Nonlinear pricing and oligopoly," *Journal of Economics and Management Strategy*, 4, 529–562.

——— (2007): "Price discrimination and competition," in *Handbook of Industrial Organization*, ed. by M. Armstrong, and R. Porter, vol. 3, chap. 34, pp. 2221–2299. Elsevier.

Sung, J. (2005): "Optimal contracts under adverse selection and moral hazard: A continuous-time approach," *Review of Financial Studies*, 18, 1021–1073.

Thanassoulis, J. (2012): "The case for intervening in banker's pay," *Journal of Finance*, 67, 849–895.

Wilson, C. (1977): "A Model of Insurance Markets with Incomplete Information," *Journal of Economic Theory*, 12, 167–207.